CONNECTING TWO DIFFERENT APPROACHES TO INQUIRY: STUDY AND RESEARCH PATHS AND STRUCTURED PROBLEM-SOLVING LESSONS

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Abstracts

This paper examines the challenges of connecting two kinds of inquiry-based didactic approaches: the study and research paths (SRPs), developed in the framework of the anthropological theory of the didactic, and the structured problem-solving lessons in the Japanese educational context. At first glance, both didactic proposals appear similar, as they emphasize the centrality of questions in instructional situations. However, a deeper analysis considering the management of the didactic time, the didactic contract, and the didactic milieu, reveals significant differences between their organizations, that is, the particular teaching patterns they promote. Furthermore, analyzing how SRPs have been adapted to the Japanese mathematics classroom also highlights ecological differences, while shedding light on some institutional constraints in the Japanese educational system that hinder their implementation by schoolteachers. Finally, our study uncovers a particular form of institutional transposition of SRPs, shaped by the specific demands of the teaching profession.

Keywords: Connecting theories; Herbartian schema; Study and research paths; Teaching profession; Archedidactic transposition.

INTRODUCTION: CONNECTING DIFFERENT APPROACHES TO INQUIRY

Over the last two decades, the networking of theories has been recognized as a crucial methodological issue in mathematics education research (e.g., Bikner-Ahsbahs & Prediger, 2014; Shinno & Mizoguchi, 2023). On the one hand, the increasing number of theories reflects the growth of the field as an academic discipline. On the other hand, further progress requires a certain coordination of this theoretical diversity. From this perspective, the networking of theories is an essential direction in the field of didactics. In this context, we use the expression "networking of theories" flexibly to encompass a broad range of meaning, as illustrated by a well-known diagram by Prediger et al. (2008). This means not only the integration of different theories but also the establishment of mutual understanding between them. Moreover, we also want to underline that the operation of "integration of theories" requires much earnest deliberation in accordance with the following principle, which we call the *respectfulness principle*:

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> "How can a theory *T* look at another theory *T'*? [...] a theory is a meticulously built construct, and that this is sufficient to outlaw any opportunistic syncretism. We must each time carefully study *T'* and then re-examine *T*—to improve it, when possible." (Chevallard, 2022a, p. 11)

Following the perspective of connecting and networking theories, this paper focuses on analyzing some didactic proposals that emerged in the framework of the anthropological theory of the didactic (ATD) (Chevallard, 2019) alongside the *structured problem-solving approach*, developed in the Japanese educational context (e.g., Takahashi, 2021). Recently, the ATD has been disseminated and is gradually gaining recognition within the mathematics education community in Japan, increasing the opportunities for Japanese teachers and educators to use some of its didactic proposals—particularly the *study and research paths* (SRPs)—in their lessons. However, establishing meaningful connections between the SRPs and *structured problem-solving lessons* (SPSLs) remains a challenge for Japanese schoolteachers and even for researchers. Thus, our research questions can be formulated in the following terms: *What are the commonalities and differences between SRPs and SPSLs? To what extent can they be effectively connected?*

Beyond practical considerations, there is also a scientific rationale for focusing on the connection of these theoretical approaches and addressing the research questions. The networking of theories in mathematics education typically involves connecting theoretical frameworks developed and used by researchers, what we can refer to as *scholarly knowledge* of didactics. However, our research approach differs as we aim to examine the (in)compatibility of a scholarly theory, established within the research community of didactics, and a professional or vocational "theory[1]" shaped within the teaching profession. In this sense, the networking of the SRP and the SPSL represents one of the most contrasting examples of a scientifically oriented approach and a practically oriented approach. As far as our analysis unfolds, the implications of this distinctive theoretical connection will become clearer, potentially opening new focuses for methodological discussion in the field of mathematics education research.

HERBARTIAN SCHEMA: A GENERIC MODEL OF INQUIRY

Due to limited space, it is not possible to sufficiently describe in depth the didactic traits of SRPs and SPSLs here, but they are revealed to some extent in the section with its comparative analysis. Besides, as basics for understanding the following sections, we offer two brief remarks from the perspective of the so-called (unstructured) *problem-solving* approach (cf. Bosch & Winsløw, 2015). Problem-solving lessons are didactic situations in which students are confronted with non-routine mathematical problems, which they solve using their own *already learned* mathematical knowledge and skills. Through the resolution process, students not only reinforce their proficiency but also learn to apply their understanding in diverse contexts. In this sense, the lessons align, *at least partially*, with the *paradigm of questioning the world* (Chevallard, 2015), where study processes are generated by the need to inquire and respond to open questions. By contrast, the SPSLs are aligned more closely with the *paradigm of visiting works* (ibid.), which is more knowledge-centered than

1 The somewhat unusual use of the term *theory* will be explained later in the subsection on metatheoretical remarks concerning the notion of theory.

traditional problem-solving lessons. Each SPSL holds a *newly encountered* mathematical content (or a "work" in the ATD's parlance), that students are expected to learn by engaging with a carefully selected set of tasks or problems. The mathematical content is first delineated beforehand, and the problems or tasks are designed accordingly to facilitate its study. With regard to SRPs, they are conceptualized in the transition towards the paradigm of questioning the world *more thoroughly* than problem-solving lessons, whereby the open questions addressed remain central and students' inquiry processes are closer to scientific researchers' ones.

In the ATD, any study and inquiry process can be described using the *Herbartian schema* (Chevallard, 2015, 2019), which models the functioning and dynamics of *didactic systems* denoted by $S(X, Y, Q)$. In this model, X represents a set of students x, Y a set of teachers y, and Q an initial *generating question* that serves as *didactic stake* for the *class* $[X, Y]$ composed of X and Y. These didactic systems are supposed to produce final answers $A^{\heartsuit}$ to Q: $S(X, Y, Q) \rightarrow A^{\heartsuit}$. Studying a question Q initiates an inquiry process involving a *didactic milieu M*, which is composed of various objects and tools that support the inquiry. In the following sections, we will specify what kinds of objects or tools are considered as part of M. A more developed form of the schema introduces the milieu component as follows: $[S(X, Y, Q) \rightleftharpoons M] \rightarrow A^{\heartsuit}$.

The Herbartian schema functions as a general reference model to analyze and design didactic processes, capable of representing any teaching and learning situation. Both SPSLs and SRPs can be regarded as particular realizations of the Herbartian schema in the following sense: they typically begin with a question posed to students and conclude with the elaboration of a final response. However, by attending to the components and dynamics that unfold the inquiry process, we can identify several significant differences between SRP and SPSLs. These differences will form the central focus of our analysis in this paper.

Before we proceed, we would like to clarify the twofold use of the Herbartian schema, which appears in this study. On the one hand, it serves as a descriptive tool by modeling the different types and dynamics of inquiry. We intend to analyze three fundamental dimensions of the inquiry with the help of the schema: (1) *chronogenesis*, relating to the temporal development of the study and inquiry processes; (2) *topogenesis*, relating to the distribution and evolution of roles and responsibilities between teachers and students; and (3) *mesogenesis*, relating to the evolution of the shared milieu by students and teachers throughout the inquiry. On the other hand, the Herbartian schema also plays a prescriptive role in designing inquiry situations. Although this function is less explicit in the present paper, it is nonetheless relevant. The analyzed SRP was, in fact, designed using this schema, which illustrates its role in structuring and guiding the design of inquiry-based didactic proposals.

A COMPARATIVE ANALYSIS OF SRPS AND SPSLS

This section conducts a comparative analysis with two selected cases by examining the three fundamental dimensions of the inquiry previously mentioned: chronogenesis, topogenesis, and mesogenesis. We begin by presenting the selection of these two cases that are representative of both approaches, and the original papers that describe both proposals. Note that in the analysis that follows, we will not consider the full picture of the chronogenetic, topogenetic, and mesogenetic aspects. Instead, based on information obtained from the

selected papers, we focus on the most significant differences between the two approaches.

Selection of SRP and SPSL cases

We have selected two representative cases of SRP and SPSL, conducted in Spain and Japan, respectively, to illustrate and analyze their key characteristics. Detailed information of these two selected cases is available in Vásquez et al. (2021) and Asami-Johansson (2021). Table 1 summarizes the components of the didactic system, *X*, *Y* and *Q*, and the initial elements included in the didactic milieu *M* shared by the students and teacher(s).

	SRP (Vásquez et al., 2021)	SPSL (Asami-Johansson, 2021)
X	Two groups of 30 students of grade 10	41 students of grade 7
Y	The first author of the selected paper acting as mathematics teacher and researcher, with two more mathematics teachers.	One experienced mathematics teacher
Q	Which padlock (among 6-7 different types) is safer?	Which of the surface area of the cones A (with 6 cm generatrix and 4 cm diameter of its bottom circle) or B (with 4 cm generatrix and 6 cm diameter of its bottom circle) is the largest?
Initial *M*	Students are given 6-7 different types of padlocks, each utilizing distinct code mechanisms, such as numerical combinations, letter sequences, directional inputs, and more. Students had not been introduced to combinatorics.	The teacher displayed figures of the two cones and their sizes on the blackboard. At the students' request, the teacher picked up a 3D cone and used a projector to demonstrate what it would look like when rotated. He then cut out the cone figures and showed what they looked like in a plane view, i.e. the net drawing.

Table 1. Didactic systems in the selected SRP and SPSL

Chronogenesis: concerning the standard amount of didactic time required

In many educational systems, didactic systems are largely shaped by the requirements of a curriculum or study programs, which delimit the knowledge to be taught and learned at each year and educational level. Within the framework of the ATD, the conditions established concerning the structure of the knowledge to be taught at schools can be analyzed through the levels of didactic codeterminacy (Chevallard, 2002): *discipline* (in our case, mathematics), *domain* in the discipline (e.g., geometry), *sector* (e.g., synthetic geometry), *theme* (e.g., triangle), and subject/*topic* (e.g., right triangle). These specific levels of codeterminacy can be used to describe the *degrees of finality* of *Q* and the instructional proposal, for *X* and *Y* (Bosch, 2019). Consequently, the extent to which an inquiry is bound by a predetermined body of knowledge can vary significantly, allowing for differing degrees of flexibility in the learning process.

When the degree of finality of *Q* is relatively low, the didactic system may generate an original final answer, which is often a *collectively created answer* denoted by $A_{[X, Y]}^{\bullet}$ which is developed within the class [*X*, *Y*], i.e. *X*, under the guidance of *Y*. Here, $A_{[X, Y]}^{\bullet}$ inclines not to be confined to a strict field of knowledge like a theme or topic, but to a relatively wide range or mixture of sectors, domains or disciplines. When, on the contrary,

there is a high degree of finality—primarily due to the predetermined nature of the topics or themes to be covered—, the elaboration of a final answer $A^{\heartsuit}$ tends to include the study of pre-existing ready-made answers $A^{\diamondsuit}$ or answers (surreptitiously) shaped by Y, which can be denoted by $A_Y^{\heartsuit}$, usually located at the level of the theme or topic.

In principle, SRPs are expected to be as much *definalized* inquiries as possible, although they are adapted to the institutional conditions and constraints of a specific educational institution. In the SRP proposed by Vásquez et al. (2021), the finality can be placed at the level of the domain or sector—specifically, in this case, within the field of combinatorics and counting:

> We rely on past results about the design and implementation of SRPs [...] and adapt them to the area of combinatorics in compulsory secondary education. We propose a SRP [*sic*] that starts from a generating question about how long it would take to open some particular types of padlocks. (p. 202)

In contrast, the design of SPSLs typically begins with the selection of a specific piece of mathematical knowledge to be developed in a lesson, starting with a relevant question. This approach results in highly structured and finalized inquiries at the topic level, as illustrated in the case analyzed by Asami-Johansson (2021):

> The title of today's lesson (and lesson plan) is "determination of the surface area of a cone". In the previous lesson, the students have learned how to determine the area of a sector of a circular disk by using the central angle a (namely $A = \pi r^2 = a/360$) [*sic*]. (p. 93: the second equal sign should be the multiplication operator.)

The degree of finality of an inquiry is also manifested in the chronogenesis by the corresponding conditions over the management of the didactic time that students and teachers devote to exploring particular questions or tasks. In many educational contexts, didactic time management is closely tied to the number of curriculum contents expected to be addressed during instruction. These contents are typically assumed to indicate the progress of the learning process.

In the case of Vásquez et al. (2021), the SRP exemplifies a long inquiry process carried out over 2–3 weeks, with four 50-minute sessions per week. The initial generating question gradually evolved into different derivative questions that structured the inquiry across sessions. As detailed in Vásquez et al. (2021), some of the key derived questions guiding this process included the following: How many codes are possible for each padlock? How can we validate that all possible codes have been accounted for? [Q_1]; Can the same techniques be used to determine the number of codes for all padlocks? [Q_2]; Is there a general formula that simplifies the counting of codes for any padlock? [Q_3]; Can the models used to count codes for each padlock be applied to other counting situations? [Q_4].

In contrast, the SPSL described by Asami-Johansson (2021) was designed and implemented within a single 50-minute lesson, beginning with the presentation of the problem and concluding with a final answer. In this context, the initial question tends to delimit the scope of the inquiry, rarely giving rise to new student-generated questions or to modifications of the initial task system. As such, structured problem-solving

activities appear to be more compatible with traditional school conditions than those involved in SRPs, and in fact, they have been developed accordingly.

Due to the greater openness of the initial question (Q_0) in SRPs and the inherent flexibility in managing didactic time—both of which allow for significant divergence based on students' questions and responses—Japanese teachers often face substantial challenges in implementing SRPs. This tension is evident in practice-oriented reports from teachers, many of which describe the adaptation of SRPs into shorter formats (e.g., Koide, 2020). We will return to this issue later in the discussion.

Topogenesis: changes in the didactic contract concerning problematization

The second dimension for comparison concerns the required changes in the *didactic contract* (in the sense of Brousseau, 1997), particularly regarding the distribution of responsibilities between teachers and students during the inquiry process. Specifically, this involves examining the clauses related to the position of *problematizer*: who is expected to pose the significant questions to be investigated?

Traditionally, the role or *topos* to pose questions has been centralized with the teachers, while students are generally expected to raise doubts or ask questions that elicit immediate answers. In contrast, in an authentic inquiry, both students and teachers can share responsibility for formulating questions about the object of study and for exploring the (possibly provisional) answers that emerge.

In line with this perspective, the role of posing problems in SRPs is ideally distributed between the students *X* and teacher(s) *Y*. In Vásquez et al. (2021), one can find, since the design of the SRP, the careful attention to the problem-posing task in the students' *topos*:

> Once the students have all the computations for the nine different padlocks [...], we expect the next following questions to appear from students, Q_3: Are there any formulas that could simplify the total counting of combinations? Are these formulas specific of the "kind of padlock" we want to understand? (p. 209)

In the SPSL examined here, the didactic contract tends to confine the responsibility for posing or selecting questions primarily to the teacher. As described in the lesson analysis (Asami-Johansson, 2021, p. 98), this responsibility falls under the teacher's *topos*:

> [The teacher] lets the students determinate the area of the circular sector A [*sic*]. While he circulates between the desks, he catches a student's murmur, "But we do not have the central angle of the sector..." [The teacher] remarks quite loudly (so that all students can hear) "The central angle? Must you have the central angle to determine the area?" Then he asks the class how many of them have the same problem. It shows the majority of them do. He comments: "Ok, you have a trouble not having the central angle. What can we do without the angle?" [...] [He] let a student M write his solution of the blackboard: $6\times6\times\pi\times1/3 = 12\pi$. Then he asks the class "Is there anyone who has a problem?" Several students raise the hands, and one utters: "How and where the 1/3 comes from?" [He] confirms the other students have the same question. (We anonymize the teacher's name in this paper. The same will apply hereinafter.)

While the students do raise some questions, the teacher typically plays the problematizing role, orchestrating much of the inquiry process. This includes actions such as posing questions, inviting the students to present their work, validating responses, and clarifying key concepts at specific moments during the lesson in order to guide the class towards convergence. These instructional gestures, led by the teacher, significantly shape the direction and possibilities of the inquiry.

Mesogenesis: concerning the evolution of the didactic milieus

In the second section, we have introduced a more developed form of the Herbartian schema, including the component of the didactic milieu *M*, denoted by: $[S(X, Y, Q) \rightsquigarrow M] \rightarrow A^{\heartsuit}$. In the sense of the ATD, a didactic milieu is any set of *works* that is possibly useful for inquiry. Here, a work is any entity, whether physical or conceptual, that is created by "someone", i.e. a person, a community, and so on. To make a long story short, let us list concrete examples of works from mathematics: concepts, operations, theorems, definitions, axioms, proofs, theories, questions, data, pieces of software, symbols, and so on.

In the schema, the leaping arrow "$\rightsquigarrow$" indicates a *procognitive* process of students' autonomous information gathering from different *media* (e.g., books, web pages, knowledgeable persons, and so on) to bring about the didactic milieu (Chevallard, 2015). This "procognitivism" of inquiry is quite different from the format of the usual didactic organizations in mathematics class, which is characterized as *retrocognitive*. There, each element of the didactic milieu is prepared or at least predetermined by the teacher. Students can only search for useful pieces of information in the inquiry from their own memories. The retrocognitive schema of inquiry with the predetermined milieu can be described as follows (Chevallard, 2007): $S(X, Y, Q; M) \rightarrow A^{\heartsuit}$.

In the case of Vásquez et al. (2021), the inquiry process fostered a procognitive dynamic in which the students continually consulted various sources of information to progress in addressing the initial question: Which type of padlock is more secure? For instance, the students used padlock simulations—either physical or digital (e.g., through Excel)—to generate datasets comprising the possible combinations for each type of padlock. As soon as they began comparing these datasets, new questions emerged: Did the simulations account for all possible combinations? Were there repetitions in the data? These questions, which also became part of the evolving didactic milieu, drove the students to refine the models they were using, ultimately guiding them toward constructing and applying numerical formulas. As the inquiry progressed and new padlock types were introduced, the students aimed to associate a mathematical model with each "family of padlocks" that shared common properties. The students consulted textbooks and online resources to find relevant combinatorial formulas, which they then sought to connect with the models they had developed for each padlock type. As Vásquez et al. (2021) report:

> All the working teams concluded with a correct answer for their padlock, since they had the complete list of combinations to check the proposed arithmetical operation, although some were more difficult than others. One team—Team D—, which had the directional padlock [...] explained to the class that they had looked up for existing answers on the web. (p. 211)

By contrast, in the case of Asami-Johansson (2021), the teacher prepared large parts of the milieu following the "retrocognitivism". The teacher had anticipated which parts the students would struggle with and

prepared several resources in advance. For instance, the teacher knew that the students would not be able to guess what the cone would look like when flattened, and he brought the geometric nets of the cones made by paper:

> [The teacher] then asks the class how the bottom of the cone looks like. It's a circle. He then picks up a circle made by a paper and puts it on the blackboard [...]. (p. 98)

The retrocognitivism is one of the general characteristics of the SPSL. This does not mean, however, that the SPSL approach belittles the so-called *adidactic* milieu within the framework of the theory of didactic situations (TDS), which encourages students to engage in autonomous mathematical activities (Brousseau, 1997). Rather, in our view, the SPSL approach has similar interests in the adidactic milieu as the TDS implies, i.e. the adidactic milieu under the constraint of retrocognitivism, even though we can find many specific differences between them. In comparison, the ATD's criterion for judging whether a given milieu is adidactic or not is much stricter: from the perspective of the Herbartian schema, the retrocognitive process itself can be a sign of the lack of adidactization of the study, even if the process is conducted by thoughtful setting up of the milieu.

Metatheoretical reflection: What is the schoolteacher's professional "theory"?

In this paper (and in the ATD as a whole), the term "theory" essentially represents an umbrella concept, as defined by the ATD:

> The word *theory* belongs to the vocabulary of the ATD, where it has a broader meaning than usual: a 3-year-old boy has a theory of dads, a man without education a theory of politics, and so on. (Chevallard, 2022a, p. 5; italics in the original.)

Therefore, a theory can be any body of knowledge, regardless of its degree of sophistication, verbalization, stability, or generality. In this sense, schoolteachers, as "researchers" in education, have their own didactic theories.

Besides, from the ATD's perspective, the word "theory" can also carry practical implication according to the following rhetorical convention:

> As usual, when we speak of "theory of...", we mobilize a *synecdoche*: a part (the theoretical component of a praxeological complex) refers to the whole (this complex itself). (ibid., p. 5; italics in the original.)

In the ATD, a "praxeological complex" or a *praxeology* is a term to describe any kind of human activity. It includes as inseparable entities a know-how or *praxis*, and a *logos* or discourse to describe and/or prescribe the *praxis*. The *praxis* is made of *types of tasks* and *techniques* to carry out the tasks, while the *logos* part includes a *technology* as an initial discourse on the *praxis* part, and a *theory* as a higher-order discourse on the technology. Due to space limitations, we cannot explain it in detail here, but it is important to note that, in its current meaning, the term "theory" (as in "number theory" or "theory of evolution") is often used not

only to refer to the "purely theoretical" elements of a given framework (i.e., technologies and theories of a praxeological organization), but also to indicate the "more practical" elements (i.e., types of tasks and techniques), that evoke and activate purely theoretical elements for their analysis and design.

Based on the two points mentioned above, in this paper, we concentrate on the more practical aspects of the whole didactic praxeologies involved in SRPs and in SPSLs, such as teaching methods and didactic processes. This is due to the gap between the "discursive grades" of their respective theoretical components. On the one hand, as a theoretical part of the ATD, the theory of SRPs is made explicit for the scientific study of didactic phenomena, especially those related to inquiry. By contrast, as a professional art refined through experience and lacking thorough mediation by theoretical discourse, the didactic praxeology of SPSLs is inherently less scientific, in other words, its theory can be described as a "pragmatic theory". This means that much theoretical part of the SPSL's didactic praxeology is expressed in scientifically less-disciplined and "teacher-friendly" terminology and logic—which is a merit of pragmatic theories—, thereby being well-embodied in teaching activity. From this perspective, we have compared practical or realized aspects of the two approaches in this section. We hope that this revealed some elements of the mainly tacit, theoretical block of the SPSL[2].

ON THE INTEGRABILITY OF THE SRP AND THE SPSL: AN ANALYSIS OF SRP-LIKE ORGANIZATIONS IN THE JAPANESE TEACHING PROFESSION

Methodological reflection: How should theoretical integrability be studied?

In the previous section, we clarified the functional commonalities and differences between the two forms of inquiry-based teaching process: the SRP and the SPSL, by analyzing two concrete examples. This comparison yields significant results from the perspective of networking theories, revealing the relationship between different didactic approaches—specifically, the SRP and the SPSL.

However, this is not the only meaningful outcome from such a perspective. Another relevant issue concerns the *integrability* of these approaches. Needless to say, this does not imply a simple combination of different theories. How can we address this issue without violating the respectfulness principle that we kept in our mind at the introduction? One possible way is to study the "acculturation" between two given theories which *proceeds spontaneously without any intention to integrate them*. As we will see below, this kind of theoretical acculturation occurs when people who live in the context of one didactic theory T (e.g., the SPSL theory) study and use another theory T^* (e.g., the SRP theory) with the aim of mastering T^*. Under such circumstances, the unintentional integration of T and T^* appears to be an inevitable part of the T-people's learning of T^*. This section addresses such a related situation concerning the SRP and the SPSL, focusing particularly on the implementation of the SRP in the Japanese educational context of the SPSL[3].

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2 Conversely, the "practical grade" of the ATD is probably quite low for most schoolteachers. The discursive grade and the practical grades of a given theory are it's-hard-to-please-everybody statuses, and it is almost impossible for a theoretical framework to satisfy both.

3 Another possible approach to theoretical integration is the *dialogue* between two theories, as it was developed in Bosch et al. (2017) for instance. In our view, however, the dialogue method is not appropriate for our case. This is again because the theory of SPSL is a theory emerged from and close to the teaching profession, whose purpose is *not* to investigate into didactic phenomena, but to facilitate the teaching of school subjects.

SRP in the Japanese teaching profession

In recent years, influenced by the work of Japanese researchers in the ATD, SRP-inspired didactic organizations have begun to find their way into the professional practice of mathematics teaching in Japan. These efforts have aimed at implementing the SRP in classroom contexts. However, as the previous analysis showed, SRPs differ significantly from typical Japanese SPSLs, particularly in terms of the three didactic dimensions: chronogenesis, topogenesis, and mesogenesis. As a result, the SRPs that are transposed into the Japanese teaching profession tend to emerge not as "pure" SRPs, but rather as hybrid forms that combine features of both the SRP and the SPSL, whether intentionally or not. In this section, we examine this hybridity while exploring the ways in which SRPs are adapted, reshaped, and integrated into existing SPSL-oriented teaching practice in Japan. This analysis sheds light on the conditions and constraints under which SRPs are implemented within different institutional and cultural contexts, as well as contributes to the broader question of how diverse inquiry-based models can be integrated into professional teaching practices.

Let us consider a representative and pioneering example of an SRP implemented in the Japanese educational context, as documented by Koide (2020). The didactic system for this SRP comprised a group of 32 8th grade students (X), the teacher-researcher (Y) (the author of Koide, 2020), and a generating question Q_0 about the *designing of a hang glider*.

The total teaching time allocated to studying Q_0 was relatively short, approximately two class periods (two 50-minutes sessions). In terms of chronogenesis, the inquiry into Q_0 quickly focused on a predefined mathematical topic: e.g., the calculation of wing areas. This shift might reflect the time constraint imposed on the inquiry. Figure 1 summarizes the questions and answers map that was followed and covered in this SRP. Regarding the topogenesis, the students occupied the problematizing topos within the inquiry process. Koide (2020) highlights one group's significant derived question: *Is the relevant wing area proportional to the pilot's body weight or not?* This question exemplifies a genuine student-led problematization within the

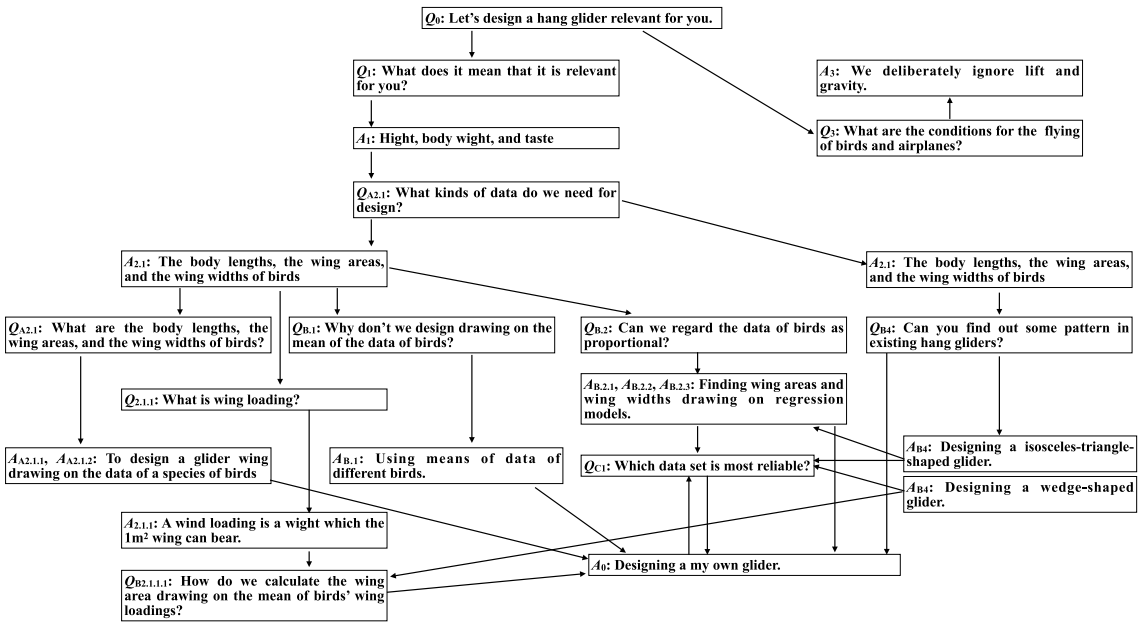

Figure 1. The Q-A map of the realized inquiry reported in Koide (2020, p. 55) (Our translation)

inquiry. Regarding the mesogenesis, the students procognitively engaged with the available resources, using the internet to gather relevant information. For example, they explored various hang glider shapes and researched the body weights of pilots suitable for each type. These efforts demonstrate the active role of the students in expanding and constructing the didactic milieu throughout the inquiry.

The SRP discussed above is part of a larger, multi-year SRP project led by the fourth author of this paper which involves in-service primary and lower secondary school teachers. The project essentially focuses on the SRP, with no explicit intention of integrating the SPSL into it, or vice versa. It has resulted in a compendium of SRPs implemented in the Japanese school context (Mizoguchi & Attached School of Tottori University, 2024). This volume documents and analyzes six SRPs, including the hang glider example introduced earlier. Table 2 presents a summary of these SRPs, organized by grades and to some of their main characteristics outlined in the previous section.

Table 2. Short summary of the SRPs in Mizoguchi & Attached school of Tottori University (2024)

Grade	Generating question Q_0	Time and finality	Students' use of media	Role of questioning
3	Each student is given a colored pen, according to their attendance number, in the order of blue, green, orange, and pink. What color is your colored pen?	4 × 45 mins (Division with remainder)	Procognitive (Permission for the use of a textbook and the internet)	[*X*, *y*] (The students share the responsibility for questioning with the teacher)
6	Let's draw the school emblem of our primary school on the school ground as accurately as possible.	13 × 45 mins (Symmetrical figure)	Procognitive (Permission for the use of a textbook and the internet)	[*X*, *y*] (The students share the responsibility for questioning with the teacher)
7	How do you win games like "The Race to 20" (Brousseau, 1997)?	2 × 50 mins (Numerical patterns for a winner strategy)	Retrocognitive (Own generation of media through the numerical simulation of the Race to 20 or the Race to *n*)	[*X*, *y*] (The students share the responsibility for questioning with the teacher)
2	How can we make a bamboo dragonfly that flies well?	4 × 45 mins (Definalized; not in Mathematics but Living Environment Studies)	Retrocognitive (The author highlights the presence of media, but it actually means students' life experience *before lessons*.)	[*X*, *y*] (The students share the responsibility for questioning with the teacher)
8	Let's design a hang glider that is relevant to you.	2 × 50 mins (Definalized)	Procognitive (Permission of the use of the internet)	[*X*, *y*] (The students share the responsibility for questioning with the teacher)
7	If all the ice in Antarctica were to melt, how much would global sea levels change?	3 × 50 mins (Definalized)	Procognitive (Permission of the use of the internet)	[*X*, *y*] (The students share the responsibility for questioning with the teacher)

Indeed, we can only get quite limited information about the SRPs implementation from the schoolteachers' reports and cannot access the raw data, but we can say, at least, that the topogenetic aspect of the SRP is relatively easy to bring in, whereas its chronogenetic aspect essentially retains the property of the SPSL. From the ecological perspective of the ATD, this chronogenetic resilience of the SPSL is conditioned by the constraints of the current educational system under which schoolteachers are involved. Let us point out here two probable constraints. The first constraint is the *disconnection of school mathematics* (García et al., 2006), which strictly structures a curricular organization as any disconnected amount of many small pieces of knowledge to be taught, whereby there is a small habitat for questions as didactic stakes within the didactic time and space instituted in advance.

The second constraint, which is related to the first, concerns the *semiprofessionalism of the teaching profession*, i.e. the fact that schoolteachers have less room to make professional decisions than "full-fledged" professions like lawyers (Chevallard, 2022b). Regarding chronogenesis, schoolteachers cannot determine the content and size of didactic stakes nor the didactic time for teaching them, as these are essentially *predetermined* by national curricula and the textbooks that they are regulated by. In other words, there is less possibility for Japanese schoolteachers to apply the chronogenesis of the SRP because national curricula have the authority to determine this. Conversely, regarding the topogenetic (and even mesogenetic) aspect, a schoolteacher may take more agency than with the chronogenetic aspect.

Feedback from SPSL to SRP: Infrastructural resources for managing SRPs

In the previous section, we discussed the hybrid modality of the SRP and the SPSL. To achieve the "sound" hybridity under the aforementioned constraints on SRP implementation, we must consider some essential elements of the SPSL organization that can be adapted for SRP implementation. Based on our observations and analysis of the SRPs of Japanese schoolteachers, we have identified at least two types of feedback related to two different levels of didactic resources in the SPSL and the SRP. First, we want to focus on the *blackboard management* as a teaching tool. In SPSLs, Japanese teachers activate a special kind of board writing called *bansho* in Japanese. The word *bansho* has the dual meaning of the process of writing board and its product (Fernandes & Yoshida, 2004; Tan, 2021). *Bansho* is a very complex fabric and has rarely been studied didactically, so that many part of it is unclear, but we can point to its two essential features at least. Firstly, it is supposed to be constructed collaboratively by the teacher y and the students X in a given didactic system $S(X, y, Q)$. Indeed, y is the main author of the *bunsho*, but y has to record and summarize X's opinions and ideas on the blackboard. Secondly, each *bansho* should function as an abstract or summary of the progress of the corresponding lesson. Accordingly, everything written on the blackboard should not be erased until the end of the lesson, especially for primary schools (Takahashi, 2006). And then, following these kinds of *bansho* principles, Japanese schoolteachers have developed various techniques for using blackboard in their lessons. So, how can teachers use blackboards to organize SRPs? Or, are there some teaching tools for SRPs other than blackboard management? These are the questions that the ATD should answer to schoolteachers to diffuse SRPs into the current teaching profession. This is the first feedback that the SPSL has given to the SRP.

The second comes from "paradidactic" resources (Winsløw, 2012), which are all resources, whether material or not, that schoolteachers can use *outside their lessons* for planning and reflection. In the Japanese teaching

profession, there is a shared writing format of lesson plans for SPSLs called *shidōan* in Japanese, which includes the aim of the lesson, the teaching process, and so on. Typically, in the lesson study process, a schoolteacher responsible for the lesson to be observed usually makes a lesson plan according to a variation of the format, distributes it to lesson observers, and then they discuss the lesson referring to the plan and the actual lesson. To increase the degree of the applicability of the SRP organization in the teaching profession, the ATD didacticians should also prepare some paradidactic tools for schoolteachers, which support their designing and analyzing their own SRPs. This is the second feedback from the SPSL to the SRP. About possible responses from the theory of SRPs to this feedback from the theory of SPSLs, one potentially promising resource from the ATD, that could complement the SPSL plan format, is the *Q-A map*, an example of which we have already seen at Figure 1. This is a tree-diagram of questions and answers that arise during the process of inquiry. For instance, Florensa et al. (2021) report the potential of the Q-A maps for secondary schoolteachers' own description of the inquiry process within the didactic organization of SRPs for teacher education. Besides, the book by Mizoguchi & Attached School of Tottori University (2024) is also supporting evidence that schoolteachers can use the Q-A maps for planning and reflecting on their own lessons.

ARCHEDIDACTIC TRANSPOSITION OF SRP IN THE TEACHING PROFESSION

In terms of the authenticity of inquiry, SRPs are much more progressive than SPSLs. However, in terms of their viability in the prevailing school systems and their infrastructures, SRPs seem to be less sustainable than SPSLs. In our view, the above Japanese realizations of SRPs are much closer to the proposal of the study and research "activities" (SRAs) (Chevallard, 2022c), which are a limited variation of SRPs but with a clear intention to construct a certain mathematical knowledge determined in advance by the teacher. In other words, SRAs are more finalized than SRPs and more limited in terms of the types of bodies of knowledge presumably needed to address its generating question. More precisely speaking, an SRA is any form of inquiry that is finalized by a piece of knowledge but maintains its procognitive nature to build up its own didactic milieu. The SRP presented in Koide (2020) fulfils this property of SRAs, although the author does not refer to the notion of SRA. In fact, the expression of SRA has hardly been introduced to the Japanese teaching profession. In the history of the ATD, the idea of SRA is a *transitional construct* that evolved into the notion of SRP (Chevallard & Strømskag, 2022). In this respect, SRAs might be less interesting than SRPs from the scientific point of view of the ATD, as the reality of SRPs is a much more revolutionary device in the transition towards the paradigm of questioning the world. But, in any case, SRAs are not contradictory to SRPs but rather indispensable to them, since SRAs can surely become parts of given SRPs when particular unknown pieces of mathematical knowledge or works might be useful for the study community involved in the SRPs (Barquero et al., 2018). All these facts imply that the SRA is a variation of the SRP relatively easy to (re)produce by teachers under the prevailing conditions and constraints. For this reason, in our opinion, the revival of the research on SRAs is a favorable condition for the diffusion of the theory of SRPs into the teaching professions of countries that assume the premises of the paradigm of visiting works.

Here we want to clarify the nature of the process of schoolteachers' adaptation of the researchers' construct of the SRP, within the framework of the didactic transposition theory (Chevallard, 1985/1991). This theory

assumes that any item of knowledge changes its structure, functioning, and utility when it changes its institutional habitat, which is different from its original producing institution. In particular, the expression "didactic transposition" indicates the transposition originating from the institutional transfer of knowledge from a *scholarly institution* to a *didactic institution*, i.e. a school system. But this is not everything. There is another form of "didactic" transposition, which has been called *archedidactic transposition* of knowledge (Artaud & Bourgade, 2022; Strømskag & Chevallard, 2024). An archedidactic transposition of knowledge signifies any institutional transposition from a producing institution to an applying institution, where the transposed knowledge is applied with a purpose or rationale that differs from its producing institution. For instance, mathematical knowledge is applied across a wide range of disciplines—such as physics, biology, engineering, economics, and psychology—each of which constitutes an applying institution that generates its own body of knowledge. These institutions spontaneously play a role for teaching and learning mathematics that are not officially alleged to be didactic but no less didactic than those found in the institutionalized educational settings. In this sense, they function as special didactic institutions referred to as *archeschools* of mathematics, where the prefix "arche-" denotes something *primordial* . This is what characterizes the transposition as *archedidactic*.

In our case, the transposition under study involves the archedidactic transposition of the theory of SRPs from the ATD research community to the Japanese teaching profession[4]. As we have seen, this transposition has been results of the study of confined SRAs rather than open SRPs in the teaching profession, that is, the transposition from the scholarly knowledge of SRPs to the *teacherly knowledge of SRAs*. Let us add here another aspect of this transposition. In Chapter 9 of Mizoguchi and Attached school of Tottori University (2024), the fourth author of this paper and some university students, who are prospective schoolteachers, propose a hybrid format of lesson plan, which is basically a variation of the usual Japanese format, but also draws on the Q-A map to describe the inquiry process. It has nine components as follows: 1) theme; 2) *a priori* analysis with Q-A map; 3) information about students; 4) reference model of inquiries; 5) background and potential of the initial question; 6) final report; 7) course structure; 8) lesson process with Q-A map; and 9) how to evaluate the students. This is an original work produced through the transposition process, created by prospective schoolteachers in terms of their own professional purpose[5].

FINAL REMARKS: TOWARD A NEW PROBLEMATIC ABOUT THEORETICAL DIVERSITY

In this contribution, we compared the approaches of the SPSL and the SRP by examining two representative examples and described the implementation of SRPs within the Japanese educational context. At a surface level, both SPSLs and SRPs are inquiry-based processes in mathematics education: they begin with an

4 In the sense of the ATD, the *teaching profession* is the professional institution of schoolteachers.

5 We interpret this transposition of the theory of SRPs as *archedidactic* rather than (purely) didactic, despite the fact that the prospective teachers are enrolled in an initial teacher education program. This is because, in this context, their aim is not merely to acquire existing skills and knowledge from the teaching profession or research community, but rather to contribute to the development of the teaching profession. The transposition thus reflects a shift in purpose—from learning established content to engaging with and applying theoretical knowledge in a new institutional setting.

interesting question and aim to guide students toward constructing an answer. However, upon closer examination, key differences emerge—differences that arise from the conditions of implementation and the institutional constraints surrounding each approach.

As our analysis has shown, this leads to an ambivalent relationship between Japanese schoolteachers and the SRP proposal. On the one hand, SRPs offer a promising instructional device, as it begins with an open generative question that is likely to interest the students. However, on the other hand, some of the conditions for authentic inquiry that the SRP model assumes appear to make its implementation in Japanese educational systems difficult. Because schoolteachers are central agents in the implementation of inquiry-based teaching, their professional knowledge and practical experience strongly influence the functioning of didactic systems. In fact, the specific characteristics of the SRPs implemented by Japanese schoolteachers appear to be influenced by their familiarity with the SPSL and its broader ecological background and context. This suggests a form of optimization or adaptation, as didactic research proposals—such as SRPs—are transposed into the professional practices of teachers. A similar phenomenon can be observed even in the Spanish case described by Vásquez et al. (2021). In that example, the SRP is indeed finalized by a relatively small school mathematics sector, combinatorics.

From the perspective of archedidactic transposition, theoretical diversity in mathematics education is not shaped solely by the theories themselves, but also by their users—whether individual practitioners or institutional actors. In our view, then, the ways in which teachers appropriate, interpret, and adapt didactic proposals derived from research constitute a central issue in understanding the broader landscape of theoretical diversity in mathematics education.

Before closing this paper, we want to give a methodological remark on the collaborative setting for the networking of theories. At first glance, this paper, written by an international team, could have involved some members responsible for the SRP, as well as others for the SPSL. However, the authors team is beyond that. All the members are active researchers in the ATD with different backgrounds (or *positions*): e.g., someone has studied the SRP since the very infant phase of the idea; someone has studied the SPSL in terms of the ATD; and someone had studied the SPSL with Japanese schoolteachers before studying it in the ATD. In the same manner, for instance, someone knows the nature of SPSL in the Japanese teaching profession well, and someone knows it as the object of study in didactics through the ATD-lens. This kind of diversity of "theoretical and practical experience" in a research team seems to be crucial for the networking of theories. It is because, according to the transposition theory, a theory itself is a changing reality depending on its institutional ecology, especially the position its creators and users take on, and whereby each researcher is only able to comprehend a limited facet of the theory. Therefore, if we are to take the aforementioned newly alleged theoretical diversity of our object of study seriously, we will probably need to establish teams that are more diverse in terms of their institutional backgrounds in order to study the networking of theories. These teams should comprise didactic researchers from variety of theoretical backgrounds, as well as schoolteachers, curriculum developers, mathematicians, and others, that is, people who are possible users of didactic theories.

Acknowledgments

First, we would like to thank the reviewers and the guest editors of this special issue for their insightful

comments on the previous version of this paper. We are also grateful to Marianna Bosch, who provided valuable feedback on our ideas before, during, and after our presentation at TSG 5.9 *"Theories in Mathematics Education"* at ICME-15. This presentation served as the foundation for the present article. The work presented is supported by the JSPS KAKENHI Grants-in-Aid for Scientific Research (18H01015 and 21K18532), as well as the PID2021-126717NB-C31 (MCIU/AEI/FEDER, UE) research projects.

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