Guest Editorial

RETHINKING THE DIVERSITY OF THEORIES IN MATHEMATICS EDUCATION. CONTRIBUTIONS RELATED TO THE TOPIC STUDY GROUP 57 OF ICME 14

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This Special Issue emerged from Topic Study Group 57 (TSG57) on the "Diversity of Theories in Mathematics Education" conducted at ICME 14 in July 2021 (Bikner-Ahsbahs et al., in print). Based on the presentations and discussions in the TSG57 at the conference, five papers have been extended and elaborated for the special issue. To introduce this special issue, we would like to look back briefly at what we have discussed in the TSG. Main topic was the Networking of Theories in mathematics education research and this also resonates with this special issue.

In recent years, the theory networking enterprise has advanced (Kidron et al. 2018). Networking of Theories means to relate different theories, to make use of them in research and scientifically reflect this process. Key assumption of this approach consists of respecting theoretical identities that are often based on cultural backgrounds and at the same time requiring from researchers to keep a critical stance on theoretical compatibility in the theoretical networking practice. To keep this balance, Prediger et al. (2008) have developed pairs of networking strategies that express increasing degrees of integration; these are *understanding other and making understandable* own theories, *comparing and contrasting* theories, *combining and coordinating* theories, and *local integration and synthesizing* of theories. Many authors in this issue recount how they used these strategies in their research.

The TSG57 addressed three subthemes in three sessions: Reasons for the diversity of theories; methodological approaches in design research related to the diversity of theories; and reconsidering specific commitments in the diversity of theories. In the first session, the TSG articulated a double aim, to respect theory development coming from various educational cultures and research areas in the field and at the same time to overcome theoretical fragmentation in the field. This requires a balance of keeping the diversity needed but simultaneously bridging theories to make communication across the theory cultures possible. Two themes were discussed, first, networking of theories can be a source of (as well as it may unpack) 'tensions' between theoretical discourses, depending on basic assumptions and, second, networking of theories can offer 'flexibility' when relating basic assumptions and thus creating a new discourse. From the second session we learned that each theory is built on the three commitments, ontology, axiology, and

epistemology, in specific ways that underlie how teaching and learning are understood, in research as well as in practice. Main postulation raised in the discussion was to increase scientific sensitivity for these commitments, specifically in terms of theoretical choices in transformative research. From the third session we have learned to make these commitments visible in research and to look for methodological ways to practice this sensitivity, as for instance by a vertical analysis grounded in historical or philosophical considerations. A crucial part of the discussion resulted in the insight that each theory on teaching and learning entails a certain understanding of ethics grounded in this theory.

Finally, two questions were raised: To what extent is it necessary or even mandatory to take ethical issues of the teaching/learning into consideration for theorizing and what are the relevant concepts of ethics for that? More generally, what are the pitfalls when such commitments go unnoticed?

This Special Issue does not address all the aspects and questions mentioned in the TSG57, but some relevant aspects are captured by the publications. All the five papers focus on and 'rethink' the diversity of theories from different perspectives. In the first contribution "Facing the Challenge of Theoretical Diversity: The Digital Case" Michèle Artigue explores the advancement in the field of technology in mathematics education in terms of research on two concepts conducted by networking of theory approaches. In the second article "The Role of a Priori Analysis in Theories", Ivy Kidron explores the notions of a priori analyses related to two theories showing how the epistemological grounding of theories determines them. In the third paper "Ethics in the Mathematics Classroom", Luis Radford elaborates the concept of communitarian ethics for the Theory of Objectification making explicit how theory and ethics are entwined. The fourth contribution "Networking Praxeologies and Theoretical Grain Sizes in Mathematics Education: Cultural Issues Illustrated by Three Examples from the Japanese Research Context" by Yusuke Shinno and Tatsuya Mizoguchi is a comparison of three Japanese networking examples that shows clearly the relevance of culture when networking theories. The final contribution by Angelika Bikner-Ahsbahs et al. is a networking case on design research leading to local integration of two theoretical approaches that allows to explain how learners transformed their behavior when acting with a digital artifact.

In the following, we are going to summarize each article and thereby extract their main contributions to the field.

SUMMARIES OF THE ARTICLES IN THE SPECIAL ISSUE

The theoretical diversity offered by different theoretical cultures is a real challenge in mathematics education research. The analysis in the paper by Artigue (2023) provides an important overview of the theoretical landscape and its main tendencies in theorizing the use of technology in mathematics education. Artigue aims to contribute to the reflection on the question: "To what extent, are we now better equipped to meet the challenge of theoretical diversity in relation to technology-based mathematics teaching and learning?" To this end, two conceptual tools are introduced in the paper. Both tools have proven their effectiveness in addressing issues of theoretical diversity: The scale of networking strategies and the concept of research praxeology (introduced in Artigue et al., 2011). Two case studies are described: The Instrumental Approach (IA) and the Documentational Approach to Didactics (DAD). The emergence and development of

these approaches illustrate well the global dynamic of the field towards increasing theoretical diversity, the questions raised by this dynamic and the insightful efforts made to deal with it. The evolution of teachers' documentational work was induced by technological advances. One important contribution of Artigue's paper is her analysis that demonstrates how knowledge has advanced both, in terms of understanding the diversity challenge and in terms of developing strategies to address it.

Kidron (2023) investigates the role of a priori analysis in different theories in the context of networking theories. A priori analysis here means an analysis, which is often carried out prior to the experiment or data collection. While the a priori analysis was initially proposed as a part of the methodology of *didactic engineering* developed within *the Theory of Didactical Situations* (TDS) (Artigue, 2020), it is today shared in different theories. As the methodology is often considered a part of the theory (Radford, 2008), the comparison of the methodologies reveal the nature of each theory as well as the relationship between them. In the previous work of networking theories, several case studies have been carried out by analyzing the common empirical data from different theoretical perspectives to shed light on their differences, commonalities, and complementarities (see Prediger & Bikner-Ahsbahs, 2014), that is to say, these studies consist of the comparative analyses of the processes and results of a posteriori analyses. Kidron's paper is unique in that it focuses on a priori analysis instead of a posteriori analysis. It shows by comparing the a priori analyses of TDS and the theory of *Abstraction in Context* (AiC), each theory has a different focus, while both theories place importance on the epistemological perspective: on the one hand, AiC focuses on the learner's construction of knowledge with a question "What is the epistemic process of the student?"; on the other hand, TDS focuses on the didactic system with a question "How this [epistemic] process is possible?" One of the important contributions of Kidron's paper is that it shows the relevance of analyzing a priori analyses in the context of networking theories. As the focus of the theory affects the data collection, the study of a priori analyses allows us to better understand the choice of data.

Radford (2023) begins his elaboration of educational ethics by an example, sensitizing readers for seeing teaching and learning as a deeply ethical form of activity. Although always present, ethics seem to have lost sight in mathematics education. Radford sees this being grounded in the conception of mathematics education that has been reduced "to a matter of acquiring knowledge, making the question of being and becoming peripheral aspects of teaching and learning" (p. 60). He traces two fundamentally distinguishable forms of ethical action back to Hobbes (1841) and Kant (2006). With reference to Hobbes, the notion of ethics consists of contractual obligations that regulate acting in society. Under the maxim of universal reason, Kant anchors moral action, hence, ethical acting, in the individual. According to Radford, Hobbes' conception of ethics informs the transmissive idea of teaching and learning, while ethics according to Kant rather inform constructivist approaches. Thus, ethics "appear framed by the way in which we understand teaching and learning" (Radford, 2023, p. 60), hence, by theories of teaching and learning. Radford elaborates the concept of communitarian ethics for the Theory of Objectification as an example. His contribution is unique as he makes explicit how the Theory of Objectification and communitarian ethics are interrelated by elaborating the theory's relational conception of the social and how this hands over to ethics. He defines communitarian ethics through three dimensions: responsibility, commitment, and care. Making use of Lévinas (1982)' understanding of responsibility he regards responsibility as "living and acting with and for others" (p. 69). The key function of communitarian commitment is "to participate in the creation of the classroom common

work" (p. 69) of joint labor. A core ingredient of educational ethics is care, meaning here that "the importance of caring for the Other is to go beyond ourselves and to be dragged powerfully into the world, to position ourselves there, with-the-Other" (p. 70). Communitarian ethics are thus relational and, consistent with the Theory of Objectification, permanently materializing themselves within the classroom.

Networking of theories is a way to bring different theoretical perspectives into a dialogue often used to unpack implicit theoretical assumptions. However, there is a lack of meta-theoretical knowledge for conducting a networking theories study. With their article, Yusuke Shinno and Tatsuya Mizoguchi (2023) want to contribute to filling this gap by answering the question *How can we characterise researchers' theoretical work on networking endeavours in terms of praxeology?* Leaning on the work of Artigue and Bosch (2014), they use the concept of praxeology introduced by the Anthropological Theory of the Didactic (Chevallard, 2019) and elaborate the concept of *grain size of theories* to characterize the researchers' theoretical work in three Japanese research and development examples that follow networking theories approaches. Their networking technique, one of the elements of praxeology, consists of networking strategies. Shinno and Mizoguchi use the strategy of comparing and contrasting theories to explore a design approach (lesson study or didactical engineering), with the strategy of combining and coordinating theories they investigate an empirical classroom study, and they use local integration for the case of a curriculum development study where the reference epistemological model (Bosch & Gascón, 2006) plays a key role. Shinno and Mizoguchi identify and compare the three networking praxeologies of the examples to describe the theoretical discourses involved. Their contribution is unique as it shows that these discourses are intrinsically related to culture, i.e., classroom culture, research culture, and curriculum culture. As reflected by the authors, these forms of culture are specific in the Japanese culture of mathematics education in that *theory for research and theory for practice* (p. 7) are distinguished.

Bikner-Ahsbahs et al. (2023) address theoretical networking between *Activity Theory* (Leontyev, 2009) and *Instrumental Approach* (Rabardel, 2002) to investigate the role of digital feedback in supporting the teaching and learning of negative numbers with a multimodal algebra learning (MAL) system. Within Activity Theory, tools such as the MAL system play an essential role as cultural objects. From the perspective of Instrumental Approach, the process of how the MAL system becomes an individual instrument can be conceived as instrumental genesis, shaped by two interrelated processes: instrumentation and instrumentalization. Bikner-Ahsbahs et al. (2023) examine the teaching and learning of negative numbers with the MAL system using Activity Theory and a model about feedback loops. They characterize the role of digital feedback as mediating between different types of action expressed by a four-layered model. One of the important contributions of Bikner-Ahsbahs et al.'s paper is their theorizing of an empirical phenomenon at the boundary of the two theories leading to 'local integration', a relatively challenging strategy of theory networking that allows to go beyond the mere understanding of a special empirical phenomenon (Prediger et al., 2008). In their networking work, after coordinating the two theories with the help of the layered model, they clearly demonstrate how local integration in terms of Radford's (2008) theory concept is achieved.

Four of the five articles are about the networking of theories, and two papers address philosophical commitments underlying educational theories. While Artigue elaborates the state of the art of research and its advancement in technology in mathematics education realized by theory networking approaches, the other three networking papers contribute directly to advance its meta-theoretical sphere. Shinno and

Mizoguchi elaborate that cultural backgrounds affect notions of theory and thus also notions of theory networking. Bikner-Ahsbahs et al. explore the role of digital feedback at the boundary of two theories as a new phenomenon and show the advancement made by the take-up of the networking strategy of local integration. Kidron, through theory networking, contributes to our understanding of epistemological commitments of theories and their methodological and conceptual consequences. Grounded in philosophical considerations, Radford improves our knowledge on ethical commitments, underlying theories on teaching/ learning in general. All the five papers advance the field by meta-theoretical considerations included into research resulting in deepening our understanding of what can make a difference in the diversity of theories in mathematics education.

Acknowledgement

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