THE ROLE OF A PRIORI ANALYSIS IN THEORIES

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Abstract

The paper offers a theoretical reflection on the role of a priori analysis in theory. For each theory the a priori analysis has a role of reference while comparing with the a posteriori analysis. Different theories offer different a priori analyses. The different a priori analyses result from the different priorities of the theories with regard to the focus of analysis. In the a priori analyses these priorities and their differences are made explicit. This study demonstrates the important role of the a priori analysis in the different theories and the benefit of comparing a priori analyses for networking the theories.

Keywords: A priori analysis, a posteriori analysis, networking theories, epistemological perspective.

INTRODUCTION

This paper offers a theoretical reflection on the role of a priori analysis in different theories. In the words "the role of a priori analysis in different theories" I mean both, the role of a priori analysis as a methodology within a given perspective and also the role of a priori analysis in the research of networking theories as a methodological tool to better understand the specificities of different theories. In the following, I explain why I consider both roles of a priori analyses, in a given theory and in networking theories.

In Kidron et al. (2018), the authors deal first with the diversity of theoretical perspectives and networking theories and only then questions concerning the role of theory are discussed with regard to the diversity of theoretical perspectives and with networking in the background. The authors explain how by means of networking theories researchers understand better what a theory is:

> In all the work related to networking theoretical approaches, deepening into the notion of theory appears as a crucial issue. (Kidron et al., p.260)

The present paper deals with the role of a priori analysis. Trying to give a general description of the term a priori analysis, we may describe an a priori analysis as an analysis, which is often carried out prior to the experiment or data collection as a part of methodology proposed within some theoretical perspective. Therefore, the answer to the question "What is an a priori analysis?" might be different for different theories.

In the present paper, I investigate the differences in the a priori analyses of some specific theories as well as the influence of the different a priori analyses on the networking of these theories.

Dealing with a priori analyses, the focus of the paper is on methodology. We will also observe how the

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notion of a priori analysis is closely related to the phase of design.

The main theories discussed in this paper are the Theory of Didactical Situations (TDS) and the theory of Abstraction in Context (AiC). There are three main reasons for the choice of these theories. The first one is that TDS as a well-established theory that belongs to the French school of didactics benefits from a strong a priori analysis. The second reason of this choice is that this paper is based on my networking experience with these theories (Kidron et al., 2008; Kidron et al., 2014). The third reason is that the a priori analyses take into account the mathematical epistemology of the given domain and for both theories the epistemological perspective is of importance.

Then, other examples of the differences in the a priori analyses of couple of theories (TDS and APC (Action, Production, Communication); CWS (Connected Working Spaces) and AiC) will be presented shortly as well as the influence of the different a priori analyses on the networking of these theories.

Only then, we will be in a better situation to understand the important role of the a priori analysis in the different theories and the benefit of comparing a priori analyses for networking theories.

A PRIORI ANALYSES IN DIFFERENT THEORIES: THEORY OF DIDACTICAL SITUATIONS (TDS) and THEORY OF ABSTRACTION in CONTEXT (AiC)

For both theories, one main focus of the a priori analysis is the epistemological perspective. We will observe how the two theories consider the epistemological dimension in different ways. TDS combines epistemological, cognitive, and didactical perspectives. TDS focuses on the epistemological potential of didactical situations. AiC analysis is essentially cognitive and focuses on the students' reasoning; mathematical meaning resides in the verticality of the knowledge constructing process and the added depth of the resulting constructs. An epistemological stance is underlying this idea of vertical reorganization (I will explain this term in more details in a next subsection). For both theories, the epistemological dimension has a significant role. In the literature, the important role of epistemology is discussed in detail in Artigue (1990, 1995). Kidron (2016) analyses the connection between epistemology and networking of theories.

In the next two subsections, I will present shortly each theory and deal with the question what is an a priori analysis for each theory. Then, I will deal with the differences of the a priori analyses and demonstrate that, in their effort of networking theories, researchers of both theories benefit of comparing their a priori analyses.

TDS a priori analysis

A short introduction to TDS is offered in Artigue et al. (2014). As mentioned earlier, this well-established theory belongs to the French school. It began to develop in the 1960s in France, initiated by Guy Brousseau (1997). In Artigue (2020, p. 203) we read "how the idea of didactical engineering which emerged in French didactics in the early 1980s contributed to firmly establish the place of design in mathematics education research". We also read that the Didactical Engineering (DE) is structured in four different phases. Design and a priori analysis is one of these four phases. Artigue (2020, p. 204) wrote: "The goal of the a priori analysis is… to build a reference with which classroom realizations will be contrasted in the a posteriori analysis". In Artigue et al. (2014, p. 48) we learn about the three characteristics of the way TDS considers the teaching and learning of mathematics. The first characteristic, the systemic perspective of TDS is expressed by the central object of the theory, the idea of situation, which is itself a system. In the present paper, a special attention is given to the second characteristic, the epistemology of mathematical knowledge, while discussing TDS a priori analysis. The third characteristic, the vision of learning as a combination of adaptation and acculturation, relates to the cognitive dimension. These characteristics determine the questions that TDS raises, as well as its methodology.

The theory is structured around the notions of a-didactical situation (in a-didactical situation there is no explicit didactical intentions: students are working as if there is no didactical intention and the teacher refrains from interfering) and didactical situations and includes concepts relevant for teaching and learning in mathematics classrooms. The social dimension also has a significant role in TDS. In essence, the central object of the theory, the situation, incorporates the idea of social interaction.

It is this systemic view that led to the concept of DE (Artigue, 1989, 2020) that we mentioned earlier in connection to TDS a priori analysis. Artigue et al. (2014, p.50) explains DE methodology:

> It is a methodology which is structured around a phase of preliminary analysis combining epistemological, cognitive, and didactical perspectives, and aiming at the understanding of the conditions and constraints to which the didactical system considered is submitted, a phase of design and a priori analysis of situations reflecting its optimization ambition; and, after the implementation, a phase of a posteriori analysis and validation.

The notion of "milieu" is an important construct in TDS. In Brousseau (1997, p.9) we read:

> Within a situation of action, everything that acts on the student or that she acts on is called the "*milieu*".

A-didactical situations are well explained in Brousseau (1997, pp, 54-72). The a-didactic milieu was initially defined by Brousseau as the system with which the student interacts in the a-didactic game. In Brousseau (1997, p.57) we read:

> The analysis of the didactical relationships implies the definition or the recognition of these "fundamental" and adidactical games, bringing together a *milieu* and a player, these games being such that knowledge – a given precise knowledge – will appear as the means of producing winning strategies".

In the design of learning situations, there is a special attention to the constituents of the milieu organized for the learner.

In her chapter "Perspectives on design research: the case of didactical engineering", Artigue (2015) presents the evolution of Didactical Engineering (DE) in the last three decades and explains its links with TDS. She also presents its characteristics as a research methodology. In this chapter we read that design has always played a fundamental role in the French school. We also read how design is connected to the a priori analysis.

In Artigue et al. (2014, pp. 54–60), we have a detailed example that explains the components of the TDS a priori analysis and the requests of the a priori analysis for developing the systemic analysis typical for TDS. For example, the need of information of the mathematical knowledge of the students, of the particular situation at stake, of the teacher's expectations regarding this situation. The methodology for analysis is described in the following sentence:

> We developed thus our analysis using the usual techniques of TDS, that is to say, preparing an a priori analysis focusing on the determination of the cognitive potential of an a-didactic interaction with the milieu, for a *generic* and *epistemic* student, that is, a student accepting the a-didactical game and able to invest in it the mathematical and instrumental knowledge supposed by the teacher. (Artigue et al., 2014, p. 63)

Therefore, in TDS a priori analysis:

– The researchers make assumptions about the supposed mathematical knowledge of the students which is required for a productive interaction with the "milieu".
– There is a need of information of the particular situation at stake, of the teacher's expectations regarding this situation.
– There is a focus on the determination of the cognitive potential of an a-didactic interaction with the milieu.
– The researchers make assumptions about the role of the teacher and how she extends the results of the a-didactical situation.

The a priori analysis must then play its role of reference as well as its role of revealing the didactic phenomena. Then the a posteriori analysis is compared to the a priori analysis and sometimes the hypotheses which were done in the a priori analysis are not in accord with the a posteriori analysis of the collected data. This comparison of the a priori analysis and the a posteriori analysis will allow the TDS researchers to deeply understand the functioning of the "situation".

AiC a priori analysis

Dreyfus & Kidron (2014) offers a short introduction to AiC. The theory is explained in more details in (Schwarz et al., 2009). AiC has been developed over the past 20 years with the purpose of providing a theoretical and methodological approach for researching, at the micro-level, learning processes in which learners construct deep structural mathematical knowledge. Methodologically (and this is the focus of the present study), the AiC researchers are offered tools that allow them to observe and analyze students' thinking processes. A detailed treatment of the methodology is offered in Dreyfus et al. (2015). AiC view of abstraction is grounded in the works of Davydov (1990) and Freudenthal (1991). I wrote earlier that AiC focuses on the students' reasoning and that mathematical meaning resides in the verticality of the knowledge constructing process and the added depth of the resulting constructs. Freudenthal ideas led his collaborators to the idea of "vertical mathematization". This idea is explained in Dreyfus et al. (2015, p. 186–187):

> Vertical mathematization points to a process that typically consists of the reorganization of previous mathematical constructs within mathematics and by mathematical means by which students construct a new abstract construct. As researchers in mathematics education, we preferred the expression "vertical reorganization" to the expression "vertical mathematization" to discern between what is intended by the teacher - the mathematization, and what often happens - a reorganization…. In vertical reorganization, previous constructs serve as building blocks in the process of constructing.

Thus, AiC defines abstraction as a process of vertically reorganizing some of the learner's previous mathematical constructs within mathematics and by mathematical means in order to lead to a new construct (for the learner). The expression used in the previous sentence *within mathematics and by mathematical means* demonstrates the importance of the epistemological dimension for AiC. For the convenience of the readers, I will report some more details about AiC. The process of abstraction has three stages: the need for a new construct, the emergence of the new construct and the consolidation of this new construct. The second stage, the emergence of the new construct is analyzed by means of three observable epistemic actions: **R**ecognizing, **B**uilding-With and **C**onstructing. Recognizing takes place when the learner recognizes that a specific knowledge construct is relevant to the problem she or he is dealing with. Building-with is an action comprising the combination of recognized knowledge elements, in order to achieve a localized goal, such as the actualization of a strategy, or a justification, or the solution of a problem. Constructing consists of integrating previous constructs by vertical mathematization to produce a new construct.

In view of AiC essential cognitive perspective, the focus is on the students' processes of construction of knowledge. In the AiC approach, contextual aspects are considered to be integral factors of the learning process. Context is regarded in a wide sense, comprising historical, physical and social context. Historical context includes students' prior learning history, physical context includes artefacts such as computers and software, and social context refers to interaction with peers, teachers and others.

Design is important for AiC. This is in accord with the epistemological stance which is underlying the idea of vertical reorganization. The design is accompanied by its epistemological aspects. As a part of the AiC methodology, an effort is made to foresee students' expected processes of construction of knowledge and an a priori analysis of the activities is conducted.

The AiC a priori analysis consists first of assumptions about the previous mathematical knowledge of the students, in particular, previous constructs which have been constructed in the past and that may be helpful in the present task. Then, the AiC a priori analysis consists of intended constructs that are required in the given task. For each intended construct, the AiC researchers give in the a priori analysis an operational definition. The operational definition will help the researchers in their decision if the student did express the intended construct. It will offer a criterium for evidence if the intended construct has been constructed. Different researchers in the team perform separately their a priori analyses. Then the a priori analyses are discussed until there is agreement between the researchers.

Like for the TDS researchers, the a priori analysis serves as a system of reference for the AiC researchers. Comparing the a priori analysis and the a posteriori analysis, the AiC researchers note that sometimes the students achieve new constructs which were not expected in the AiC a priori analyses. This fact is an important and interesting stage in the research. Sometimes, students only achieve constructs that partially match a corresponding intended construct in the a priori analysis (Ron et al., 2010).

As explained in Dreyfus et al. (2015), the AiC a priori analysis is not only a list of intended constructs. It is more a structure of intended constructs with some interactions between the different constructs. For example, some constructs are contained in others. Some intended constructs might be a prerequisite for others. Sometimes, possible paths of thinking are taken into account. This is relevant, for example, for a priori analyses of justification tasks. Justification is a specific case of construction of knowledge. Each itinerary of thinking towards the justification might be in itself a kind of construction of knowledge and different itineraries of thinking, each with a structure of intended constructs might appear in the a priori analysis.

Comparing the a priori analyses of TDS and AiC and the benefit for networking

The two theories share the same aim: to understand the epistemological nature of the episode but different questions are asked:

AiC: What is the epistemic process of the student?

TDS: How this process is possible?

For both theories, TDS and AiC, the epistemological perspective is of importance but their a priori analyses have a different focus. In AiC the focus of the a priori analysis is on the learner's construction of knowledge. The a priori analysis reveals hypotheses about constructs that might be observed during the construction process. For AiC, processes of abstraction are inseparable from the context in which they occur. The notion of context is very wide in AiC. The context has many components. For example, as mentioned earlier, the task, the computer, the teacher, the social interaction between students are considered as part of the context. The AiC a priori analysis with its focus on the learner's processes of construction of knowledge cannot take explicitly into account all the contextual factors. In a later phase, the researchers will analyze the influence of the context on the construction processes that were observed in the analysis of data. For example, Kidron & Dreyfus (2010) analyzed the influence of the computer on the construction processes observed in the analysis of data. They describe how instrumentation led to cognitive constructions and how the roles of the learner and the CAS intertwine. But there is an essential difference if the researcher analyzes her data taking explicitly into account in advance the contextual factors or if she first analyzes the data and the processes of construction of knowledge and only then she analyzes the influence of the contextual factors on the construction processes.

For TDS, the situation is different. The focus is on didactical systems. TDS observes the entire situation and not only the student and the mathematical activity. For example, TDS is interested in relations between systems and the teacher is an element of the system. Consequently, TDS considers already in the a priori analysis the role of the teacher and how he extends the results of the a-didactical situation. Because of the different foci between TDS and AiC, context is not theorized and treated in the same way in the different theories. This fact has an important consequence on the differences of the a priori analyses.

AiC a posteriori analysis might be influenced by the fact that some contextual factors are not taken explicitly into account in the a priori analysis. As a consequence, some excerpts which might add direct knowledge in the analysis of the cognitive processes might be missed if one focuses first on the cognitive processes and only then analyzes the influence of other parts of the context.

Kidron et al. (2014) refer to a networking case that links three theories. The issue of context is compared and contrasted in the three theories. The analyses from the different perspectives refer to a set of data from a video recording that show a session from the group-work of two students, during a teaching experiment on the exponential function in secondary school.

In Kidron et al. (2014, p. 175), the authors noted that

> An interesting, and also revealing, point is the fact that, in the analysis, AiC researchers focus on the autonomous work of the students, while TDS researchers pay more attention to the episode where the students interact with the teacher...

The a posteriori analyses of the two theories are influenced by the differences in the a priori analyses and their different priorities in their focus of analysis. Different units of analysis are considered and as a consequence of the focus of analysis, as demonstrated in the a priori analysis, each theory shapes the kind of data that is appropriate to this focus. As pointed by Radford (2008):

> ...it is through a methodological design that data is first produced; then the methodology helps the researcher to “select” some data among the data that was produced but also helps the researcher to “forget” or to leave some other data unattended.

As a consequence, the different a posteriori analyses conducted within the two theories complement each other. Each analysis highlights a specific view which reflects the focus of research of the given theory. AiC analysis, with its specific tools, offers a fine-grained analysis of the students constructing processes. TDS, with its different focus, analyses the entire situation and, in particular, the interaction between the teacher and students. For example, in Kidron et al. (2014, p. 172) we read how TDS analyses the role of the teacher:

> TDS complements the AiC analysis in analyzing how the teacher extends the outcomes of the a-didactical interaction. The TDS analysis seems to start where the AiC analysis stops.

The different a priori analyses result from the different priorities of the theories with regard to the focus of analysis. Investigating these differences in the a priori analyses might lead to a better understanding of the different a posteriori analyses and to the insights offered by one theory to the other one in the networking process. I will extend this comment in the concluding remarks of the paper.

A PRIORI ANALYSES IN DIFFERENT THEORIES: CONNECTED WORKING SPACES (CWS) and ABSTRACTION in CONTEXT (AiC)

Psycharis et al. (2021) describes a research study in which students experience functions in a plurality of settings: physical context, geometry, measures, algebra. Two frameworks are used: Connected Working Spaces (CWS) (Minh & Lagrange, 2016) and AiC. CWS builds on the idea of “Mathematical Working Spaces” (MWS). The MWS theory is well described in Kuzniak et al. (2016) and more recently in Kuzniak et al. (2022).

Psycharis et al. (2021) wrote

> Work in a MWS is organised around three dimensions: semiotic (symbol use, graphics, concrete objects understood as signs); instrumental (construction using artefacts, such as geometric figure, graphs etc.) and discursive (justification and proof using a theoretical frame of reference). CWS builds on this idea of MWS by considering that in activities involving mathematics and other settings, students have to work in several working spaces and to coordinate the semiotic, instrumental and discursive dimensions of these spaces.

In this specific research study, the students had to work in several working spaces and to coordinate the semiotic, instrumental and discursive dimensions of these spaces. This is an example of networking that begins already in the networking of the a priori analyses. The a priori analyses are different in the sense that they complement each other. CWS a priori analysis identifies the different working spaces and the three dimensions (semiotic, instrumental, discursive) in each of the working spaces as well as the opportunity for the students to make connections between the working spaces. AiC a priori analysis relies on the structure offered by CWS. CWS alone could not offer, in its a priori analysis, maximal assumptions on learning. AiC a priori analysis is an effort in this direction, especially which constructs might be observed during the process of knowledge construction. In Psycharis et al. (2021) we read:

> CWS alone allows merely minimal assumptions regarding learning, but it provides a reliable structure for more precise hypotheses by AiC.

In this example of networking between CWS and AiC, we observe how comparing a priori analyses might enable to support the communication between the two different theoretical approaches. The explicit differences between the two a priori analyses demonstrate how CWS combines well with AiC and how connections between working spaces contribute to conceptualization.

A PRIORI ANALYSES IN DIFFERENT THEORIES: THEORY of DIDACTICAL SITUATIONS (TDS) and APC (ACTION, PRODUCTION, and COMMUNICATION)

Artigue (2021) describes how the two theoretical frames TDS and Action, Production, and Communication (APC) (Arzarello, 2008) were used in the thesis of Michela Maschietto (Maschietto, 2002).

Arzarello and Sabena (2014) offer a short introduction to the APC theory, especially the fact that APC provides a frame for investigating semiotic resources in the classroom. The authors describe the importance of gestures for communication and thinking. They underline that the main components of the APC-space are the body, the physical world, and the cultural environment and cite Arzarello (2008, p. 162):

> The APC-space is built up in the classroom as a dynamic single system, where the different components are integrated with each other into a whole unit. The integration is a product of the interactions among pupils, the mediation of the teacher and possibly the interactions with artifacts. The three letters A, P, C illustrate its dynamic features, namely the fact that three main components characterize learning mathematics: students' actions and interactions, their productions and communication aspects.

We also read that "space" should not be considered as a physical entity, but rather in an abstract way.

Artigue describes the tensions that appear comparing the different a priori analyses offered by TDS and APC. The theories belong to different cultures but, as in the case of TDS and AiC, for both the epistemological dimension is important. The main problem was how to create a DE that suits both theories, TDS and APC. I translate freely how Artigue (2021, p. 37) points to the main source of tension between the two a priori analyses:

> In a didactic engineering consistent with APC theory, the gestures of the students expressing cognitive constructions, the way in which these gestures will be taken up and exploited by the teacher through semiotic games, are essential ingredients of the cognitive dynamics of the class. They escape the potential for anticipation and control of the trajectories of a priori analysis.

The interesting part is that, in this specific context, to overcome this source of tensions and in order to be consistent with the Italian culture, the notion of a priori analysis was revisited, especially for what concerns the role and interaction of the teacher in semiotic games with the students. After performing these revisions, the DE that resulted was perfectly successful for this research.

CONCLUDING REMARKS

Considering each specific theory alone, we observe the role of reference of the a priori analysis while comparing with the a posteriori analysis. Comparing the different a priori analyses offered in different theories, we better understand the role of a priori analysis. In the present paper, the focus on comparing a priori analyses highlights the important role of a priori analyses in the networking of theories.

Kidron et al. (2018) wrote that in the last fifteen years:

> Different aims in the efforts to network theories were differentiated. In some cases, the researchers were interested in the complementary insights that are offered when given data or an empirical phenomenon is analysed with different theories. In other cases, the interest in the rich diversity of theories was to explore the insights offered by one theory to the other.

This idea appeared already in Arzarello et al. (2007, pp. 1625-1626) in which examples of different profiles of networking were presented.

Comparing a priori analyses, the aim is to explore the insights offered by each theoretical lense to the other from the very beginning. The networking of theories begins already in the stage of a priori analysis and not only for comparing a posteriori analyses.

We wrote that the different a priori analyses result from the different priorities of the theories with regard to the focus of analysis. In the a priori analyses these priorities and their differences are made explicit.

Reflecting on the role of a priori analysis in both theories, TDS and AiC, we realize its importance and why it is necessary towards a better understanding of the a posteriori analysis of the collected data. For both theories, the a priori analysis plays a role of reference while comparing the a priori and a posteriori analysis. For TDS, it plays the role of revealing the didactic phenomena and helps to deeply understand the functioning of the "situation". For AiC, it offers a structure of intended constructs that are required in a given task as well as possible paths of thinking. We also realize the importance that each theory keeps the specific characteristics of its a priori analysis. I wrote in a previous section that there is an essential difference if the researcher analyzes her data taking into account in advance the contextual factors or if she first analyzes the data and the processes of construction of knowledge and only then she analyzes the influence of the contextual factors on the construction processes. This essential difference is tightly connected to the specific characteristics of the different a priori analyses for AiC and TDS. In Kidron et al. (2008, p. 262), we read that:

> In networking, we want to retain the specificity of each theoretical framework with its basic assumptions, and at the same time profit from combining the different theoretical lenses. What we aim at is to develop meta-theoretical tools able to support the communication between different theoretical languages, which enable researchers to benefit from their complementarities.

Comparing a priori analyses might enable to support the communication between different theoretical approaches:

Realizing some common points in the a priori analyses enables the beginning of a dialogue between the theories. In the case of networking between TDS and AiC, for example, the common points in the epistemological dimension help towards the beginning of the dialogue (as demonstrated in Kidron et al., 2014). This idea could be used for other theories and other cases of networking: Some other common points in the a priori analyses, for example, the social dimension, might help towards the beginning of the dialogue.

Realizing the differences in TDS and AiC a priori analyses, we better understand the choices of data (as well as the "data which was left unattended") that researchers of each theory select for a posteriori analyses. Sometimes, the data which was left unattended by one theory might add direct knowledge in the analysis of this theory and, as a consequence, some cognitive processes, for example, might be missed. The complementary insights which are missing might be offered by means of networking theories. This situation might happen in different cases of networking theories.

Also, for TDS and APC, the epistemological perspective, as a common point in the a priori analyses, enables the beginning of a dialogue. In spite of the differences and tensions between the two cultures a DE was created that suits both theories and was successful for the research study. In the case of CWS and AiC the explicit differences in the a priori analyses led to the understanding that the a priori analysis of AiC relies on the structure offered by the a priori analysis of CWS. The two theories complement well each other, and the networking experience begun at the stage of comparing a priori analyses.

As we noted in AiC and APC, "a priori analysis" exists outside the French context but with different meanings in different theories.

Moreover, there are theories with which the researchers do not carry a priori analysis. Even so, each theoretical approach, as a research methodology, is used with task sequences that have been designed with well-defined conceptual learning objectives in mind. Sometimes, the data required in order to do the appropriate analysis from the point of view of a specific theory might be different than the data required in another theory. Reflecting on these differences might allow a beginning of dialogue between theories.

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