

# FACING THE CHALLENGE OF THEORETICAL DIVERSITY: THE DIGITAL CASE

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## **Abstract**

The presentation of TSG57 at ICME-14 outlined that mathematics education is a scientific field in which many theoretical cultures coexist, and that “this diversity can be regarded as richness but it also challenges research as well as communication and cooperation in the field”. It was added that “how the scientific community can cope with this diversity with scientific integrity remains an open question”. In this paper, we propose to contribute to the reflection on this challenging issue by considering research on technology-based teaching and learning. First, we present a brief overview of the theoretical landscape in this area of research, highlighting its diversity. We then introduce two conceptual tools that have proven their effectiveness in addressing issues of theoretical diversity: the scale of networking strategies and the concept of research praxeology, before focusing on two case studies. These regard the instrumental approach and the documental approach to didactics, the emergence and development of which illustrate well the global dynamic of the field towards increasing theoretical diversity, the questions raised by this dynamic and the insightful efforts made to deal with it.

*Keywords:* mathematics education, digital technology, research praxeology, scale of networking strategies, instrumental approach, documental approach to didactics

## **1. INTRODUCTION**

Increasing theoretical diversity is a general phenomenon in mathematics education, and research focused on technological issues is no exception, as we already showed it in our plenary lecture at CERME5 fifteen years ago (Artigue, 2007). In fact, in that lecture we pointed out that, for decades, this area of research has reflected the general trends and major developments in the field but has also been a source of inspiration for it, as shown for instance, by the contribution of research in this area to constructivist and situated perspectives as well as to semiotic and embodied approaches. We also argued that such a situation made this area of research an interesting window through which to look at theoretical diversity and at its practical implications. This is all the more so as the development and the use of digital technologies, since their emergence, have been given a transformative aim in mathematics education.

A similar vision is proposed in the volume resulting from ICMI Study 17, the second ICMI Study dedicated to the teaching and learning of mathematics with digital technologies (Hoyles & Lagrange, 2010). In fact, in this volume, Chapter 7 (Drijvers et al., 2010) is especially devoted to theoretical perspectives. The

authors point to the influence on this area of research of the global evolution of the field from constructivist to socio-cultural perspectives, and they illustrate this with three examples: the webbing and situated abstraction frame in reference to (Noss & Hoyles, 1996), the theory of didactical situations and the related notion of “antagonist milieu” (Brousseau, 1998) and the perceptuo-motor activity frame developed by Nemirovski for use in research on modelling environments involving physical apparatus, such as the well-known water wheel (Rasmussen & Nemirovski, 2003). They also highlight the theoretical creativity of this area of research. Two “current developments” are then presented with more details: the instrumental approach on the one hand, and mediation and semiotic mediation on the other hand. In the first case, the authors rightly point out the fact that the theoretical combination at stake between cognitive ergonomics and the anthropological theory of the didactic has led to different variations reflecting the particular weight given to these two perspectives, the first one cognitive, the second one institutional (see Section 3 for more details), although all variations share fundamental distinctions and concepts, such as the distinction between artefact and instrument, and the concept of instrumental genesis. For the second, the researchers involved share the claim that the epistemological nature of mathematical objects makes them accessible only through the mediation of representations, and that technological devices substantially alter these mediation possibilities. However, once again, as the authors point out, these premises are dealt with through different theoretical constructions, for instance the concept of human-with-media introduced by Borba and Villareal (2005), which emphasizes the unity between humans and tools, or the theory of semiotic mediation (Bartolini Bussi & Mariotti, 2008), which emphasizes the essential role of the teacher in the transition from personal meanings rooted in the context of the artefact to mathematical meanings. Despite the fact that the chapter begins with the assertion that, due to technological evolution, “communication has become a more integrated part of technology use”(p. 98), the theoretical influence of this evolution does not seem very strong at the time, rather a line for future research. Looking to the future, the authors write that the key word should be “connectivity”, because connectivity is a key word for approaching a technological evolution that is fundamentally changing our modes of communication and learning opportunities in and out-of-school. In fact, this issue is mainly addressed in Chapter 11 (Beatty & Geiger, 2010), where it is emphasized that technological evolution calls for theoretical frameworks that allow researchers to approach mathematical learning as a collaborative endeavor in technologically enhanced communities of practice. Various examples are presented where the influence of theoretical constructs associated with communities of practice (Wenger, 1998) is clear. However, there is no doubt that at the time of the ICMI Study 17 research on these newer technological affordances was not so much developed.

The chapter on technology in the book published for the 20th anniversary of ERME (Dreyfus et al., 2018) confirms the theoretical diversity observed in the ICMI Study volume (Hoyles & Lagrange, 2010), and organizes it in relation to a didactic tetrahedron with knowledge, technology, students and teacher as edges (Trgalová et al., 2018). However, it does not show an obvious movement of research towards the latest technologies. In fact, the evolution observed in the last decade, at least until recently, seems to be mainly due to the increase of research on teachers, their knowledge and their practices in digital environments, their preparation and professional development, their documentational work. For example, the documentational approach to didactics began to develop about fifteen years ago, motivated both by this attention to teachers and by the evolution of their documentational work induced by technological advances, as well explained in

(Gueudet & Trouche, 2010). This increased attention is especially addressed in the book *The Mathematics Teacher in the Digital Era* (Clark-Wilson et al., 2014) that makes clear the variety of theoretical constructs developed to address these issues. These include the adaptation of Ball's mathematical knowledge for teaching model ((MKT) to the technological pedagogical and content knowledge model (TPACK) by Mishra and Koehler (2006), Ruthven's structures features of classroom practice framework (Ruthven, 2007), and the extension of the instrumental approach to the teacher, leading to specific constructs such as instrumental orchestration (Trouche, 2005), double instrumental genesis (Haspekian, 2011) and genesis of use (Abboud & Vandebrouck, 2013). They also include, of course, the specific constructs associated with the documentational approach, such as the concepts of documentational genesis, resource system and documentational trajectory (for an updated vision, see Trouche, Gueudet & Pepin, 2019). Of particular interest for a reflection about networking is the chapter written by Ruthven, which explores the similarities, complementarities and contrasts between instrumental orchestration, TPACK and his own framework (Ruthven, 2014). Another point worth highlighting is the place given to the instrumental approach and its extensions in these various syntheses. For instance, in (Clark-Wilson et al., 2014) an entire section with six chapters is dedicated to related research.

In recent years, however, the context has been transformed by major technological changes: the massification of mobile multi-touch technologies, the development of virtual or augmented reality devices, and the growing influence of social networks on practices. The context has also been transformed by the disruptions caused by the pandemic situation, with the abrupt shift to online or hybrid forms of teaching at all levels of education, and the efforts made to limit the growth of educational inequalities by investing massively in the most widely accessible technologies, such as mobile phones. Actions and research are multiplying in response to the new challenges encountered, which will undoubtedly add to the existing theoretical diversity.

In the aforementioned CERME lecture (Artigue, 2007), we argued for the collective development of networking activities in order to limit the growing risk of fragmentation of the field. This need is also stressed in the ICMI Study 17 volume where it is written that the key word for the future should be "connectivity", not only for reasons of technological evolution, but also because efforts should be made to better connect the existing diversity of theoretical perspectives. This is even more necessary today. Since then, however, the situation has changed with the undeniable advances of the networking enterprise (Bikner-Ahsbahs & Prediger, 2014; Kidron et al., 2018). These are both conceptual, methodological and practical. To what extent, then, are we now better equipped to meet the challenge of theoretical diversity in relation to technology-based mathematics teaching and learning? Our aim in this paper is to contribute to the reflection on this question. To this end, we introduce two conceptual tools in the next section before exploiting them to two case studies. Over the last decade, we have indeed experienced the strong potential offered by the combined use of these two conceptual tools to address the challenges posed by theoretical and linguistic diversity (Artigue, 2019, 2021; Artigue & Bosch, 2014; Mesiti et al., 2021).

## 2. INTRODUCING TWO CONCEPTUAL TOOLS

### 2.1. The scale of networking strategies

As explained in (Bikner-Ahsbahs & Prediger, 2010), this construct aims to show the variety of strategies that can be used to create connections between theories. The scale distinguishes eight strategies, paired and ordered between two extreme positions: “ignoring other theories” and “unifying globally”, both of which considered not desirable. However, as the authors themselves point out, this idea of a linear order must be taken flexibly, as the degree of integration depends on the precise work done, not just on the strategy.

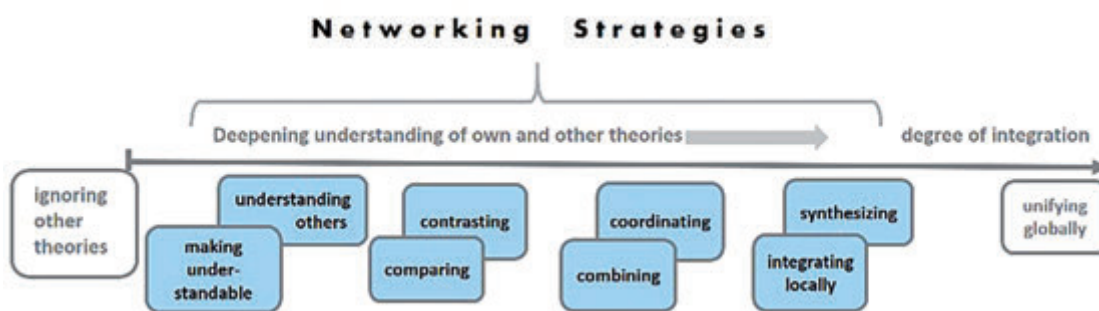


Figure 1: Scale of networking strategies (Bikner-Ahsbahs, 2016, p. 34, adapted from (Bikner-Ahsbahs & Prediger, 2010, p. 492) (CC BY 4.0))

The authors provide precise definitions for these different strategies. For example, on page 495, they explain that coordinating means that “a conceptual framework is built by well-fitting elements from different theories” and that this strategy therefore presupposes the complementarity of the theoretical approaches involved, whereas combining means that “the theoretical approaches are only juxtaposed according to a specific aspect”. The combining strategy can thus involve theories with some conflicting basic assumptions. On page 496, we are told that integrating locally and synthesizing label a pair of strategies that focus “on the development of theories by putting together a small number of theories or theoretical approaches into a new framework”, and the authors differentiate between the two strategies by considering the dissymmetry/symmetry of the theories involved in terms of scope and degree of development. As explained in (Artigue 2019), we find it more appropriate to differentiate between these two strategies according to the dissymmetrical/symmetrical contribution of the theories involved in the resulting construction, and to consider that any time there is a significant dissymmetry in the connection, even for theories having a similar state of development, there is local integration. This is the case, for example, when a broad theory is enriched by theoretical constructs from another well-established theory in order to build an integrated theoretical framework that takes into account research findings in a particular area produced in a different theoretical culture.

### 2.2. The concept of research praxeology

The notion of research praxeology introduced in (Artigue et al., 2011) extends to research practices the concept of praxeology at the heart of the anthropological theory of the didactic (hereafter ATD) (Chevallard,

2019). Indeed, a basic principle of ATD is that all human practices can be modelled in terms of praxeologies. These consist of a praxis block and a theoretical block in dialectical interaction. A priori, thus, this is also the case for research practices. By definition, the praxis block of research praxeologies includes the different types of tasks that the research activity requires, and the associated techniques of study. Their theoretical block consists of the discourse used to describe, justify and interpret research techniques (technological discourse in ATD language), and a theory consisting of “statements of a more general and abstract character, with a generally strong justifying and generating power.” (Bosch & Chevallard, 2020). At the simplest level of point praxeologies, dealing with a single type of task, a typical research praxeology in mathematics education is associated with a research question and a technique of study for this question. These form its praxis block. This block is dialectically linked to a theoretical block consisting of technological and theoretical discourses. The theoretical block includes at least some methodological discourse that explains and justifies the technical choices; some research background, namely existing knowledge regarding the question at hand or related questions, and associated constructs that are considered of interest, plus at least one theory that underlies the whole construction.

Of course, this is a very simple case. Most research questions cannot be solved by using a single technique, and the theoretical block of research praxeologies, even point praxeologies, often, does not mobilize a single theory but some theoretical combination. Most often, research praxeologies emerge from questions about teaching and learning processes, or more globally about the functioning of didactic systems or institutions, that is, through their praxis block. But the questions and their formulation are influenced by the “theoretical” already present in the researcher’s or the research team’s environment. The study techniques are guided by those that are implemented in close research praxeologies, or that are familiar to the researcher. In this way, the praxis and the theoretical blocks interact from the outset. Moreover, the study of a research question generally requires more than a single point praxeology. It is necessary to develop a coherent set of research praxeologies that share, at least in part, the same theoretical block. Local and regional praxeologies thus emerge. New questions also arise from research results; new elements enter the technological discourse in terms of distinctions, categorizations, didactic phenomena. If they become reasonably shared, they will enrich the theory itself, thus contributing to the dynamic of research praxeologies.

We claim that approaching the relationships between theories through this praxeological lens can help researchers to take better account of the functional role of theories in the research activity. The interest in adopting such a functional stance towards theories became clear to us while working in the European project TELMA whose aim was to capitalize European knowledge about technology enhanced learning in mathematics (Artigue, 2009). At that time, however, the concept of research praxeology had not yet been introduced. In the context of research on the educational use of digital technologies, its first use took place in a retrospective analysis of the theoretical activities carried out in the ReMath European project that had emerged from TELMA (Artigue & Mariotti, 2014).

In the following two sections, we use these tools to present and discuss two case studies. These regard the instrumental approach and the documentational approach to didactics, respectively. Both approaches have obvious connections and both have generated a multiplicity of theoretical interactions, given the diversity of researchers who have contributed to their development or simply used them. They provide a limited, but sufficiently rich, ground to support the reflection aimed at in this article.

### 3. A FIRST CASE STUDY: THE INSTRUMENTAL APPROACH

#### 3.1. Overview of the instrumental approach

The instrumental approach (IA in the following) emerged in France in the mid-1990s in a specific technological context, that of CAS technology (Artigue, 2002), and with two theoretical pillars: the ergonomic perspective on contemporary tools developed by Rabardel and VÉrillon (Rabardel, 1995; VÉrillon & Rabardel, 1995) and ATD<sup>1</sup>. In fact, this approach quickly spread beyond the small community in which it was born. The rapid publication of books, both in French and in English, and of many articles certainly favored this dissemination. All over the world, researchers began to incorporate IA constructs into their theoretical frameworks. This phenomenon was already evident at the time of the ICMI Study 17 mentioned above, and in the last decade the theoretical diversity has even increased. All constructions share the reference to the Rabardel's and VÉrillon's ergonomic perspective, in particular:

- the distinction between artefact and instrument;
- the concept of instrumental genesis with its two movements dialectically connected: instrumentalization from the user to the artefact and instrumentation from the artefact to the user;
- and the conceptualization of instrumental genesis in terms of the elaboration or appropriation of schemes.

However, these constructs are combined with or integrated into different theories, which leads to significantly different research praxeologies. This phenomenon was already visible in the first two doctoral theses based on IA, those of Defouad (2000) and Trouche (1997), the latter having the theory of conceptual fields (Vergnaud, 1991, 2009) as its main theoretical component. The impact on the different components of their respective research praxeologies is clear, with obvious consequences on the results obtained. In Defouad's thesis, in line with the ATD conceptualization of human activities in terms of praxeologies, more emphasis is placed on the instrumented techniques developed, the material signs (ostensives in ATD language) used in them, and the discourse explaining and justifying the techniques (the technological discourse in the praxeological model), rather than on the schemes underlying them. Furthermore, special attention is paid to the relationships between the students' instrumental geneses and the classroom instrumental genesis, and their management by the teacher. The results are mainly expressed in terms of regularities identified in the students' instrumental geneses and of didactic phenomena that show links between the characteristics of the students' instrumental geneses and the institutional conditions and constraints that shape the ecology of instrumented techniques in the classroom. In Trouche's thesis, the results are mainly expressed in terms of schemes of instrumented action and of their evolution in the transition from graphic to symbolic calculators.

A few years later, in the ReMath project (Kynigos & Lagrange, 2014), thanks to the development of specific methodological tools, we were able to analyze the effect on research praxeologies of the integration of instrumental perspectives in theories born outside the French didactic tradition such as the theory of semiotic mediation (Bartolini Bussi & Mariotti, 2008) or constructionism (Papert, 1980). New local integrations have also resulted from the extension of IA to the teacher, such as its integration in the dual

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<sup>1</sup> More detailed information about the instrumental approach, its emergence and development, can be found in the Michèle Artigue's Unit of the ICMI AMOR project: <https://www.mathunion.org/icmi/awards/amor/michele-artigue-unit>

ergonomic and didactic approach of teachers' practices (Robert and Rogalski, 2002), well illustrated by the GUPTEN project in France (Lagrange, 2013).

### **3.2. *Networking challenges: the scheme/technique case***

There is no doubt that all these connections have contributed to the wealth of knowledge and results produced under the IA umbrella, but they have also raised serious challenges. We will illustrate these challenges with an example, the scheme/technique case, which has been the source of intense debate for almost a decade (see the synthesis offered in (Monaghan, 2007)). IA emerged as a synthesis between Rabardel's and Vérillon's instrumental perspective and ATD. In such a process, ATD categories and discourses were intertwined with those proposed by cognitive ergonomics. Instrumental geneses were thought in terms of the development of praxeologies, and thus approached in terms of instrumented techniques and associated technological discourses, rather than only in terms of schemes, as was the case in ergonomic publications. This duality was discussed extensively in the first decade of IA. The collective work on this issue helped to clarify the different points of view and, in particular, to reject simplistic assimilations such as those that reduced techniques to gestures or to the observable part of schemes. In fact, thinking in terms of praxeologies means that techniques cannot be isolated from the technological discourse that describes, explains and justifies them. In a sense, reducing techniques to gestures is akin to reducing schemes to their observable characteristics without considering the essential component of schemes that the operational invariants underlying the observed regularities are. Indeed, the many contributions to the scheme/technique debate have made it clear that schemes and techniques correspond to two different and complementary ways of approaching instrumental issues, both insightful but irreducible to each other.

However, a new element deserves to be taken into account in today's reflection on this topic. In fact, while maintaining its institutional anchorage, ATD has progressively incorporated the concept of personal (mathematics) praxeology, following research by Crozet and Chaachoua (2016)<sup>2</sup>. Like institutional praxeologies, personal praxeologies are modelled in terms of quadruplets that dialectically articulate a praxis block consisting of types of tasks and techniques and a theoretical discursive block. However, this personal dimension, the very nature of the students' verbalizations used to identify the theoretical block of personal praxeologies, should allow for more productive connections between the language of schemes and the praxeological language.

### **3.3. *The ReMath contribution***

The European ReMath project also had important spin-offs in terms of networking involving IA, as it soon became clear that, despite the diversity of their theoretical cultures, five of the six teams involved were sensitive to Rabardel's and Vérillon's conceptualizations and had incorporated them in some way into their theoretical background. Thanks to a carefully designed methodology, we were able to document and analyze the effects of these incorporations both in the design and development of digital dynamic artefacts (DDAs) – six DDAs were actually created or improved as part of this project – and in the use of these DDAs by two different teams, the team in charge of the DDA design and another team from another country with a

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2 As explained in Module 9 of the Artigue's unit of the AMOR project, the idea of personal praxeology had been already introduced in Defouad's thesis, but it did not disseminate at the time and was soon forgotten.

different theoretical background. In this paper, we focus on the case of constructionism, and a comparison involving the Greek team in the Educational Technology Lab (ETL), for which constructionism was the main theoretical support, and the French team in DIDIREM, now the Laboratory of Didactics André Revuz (LDAR), whose theoretical background combined IA with the theory of didactic situations (Brousseau, 1998) and Duval's semiotic theory (Duval, 1995), both very influential in French didactic research on digital technologies (Artigue & Trouche, 2021). As explained in the special issue (Kynigos & Lagrange, 2014), in ReMath ETL was in charge of two DDAs, Cruislet and MaLT, and DIDIREM was in charge of the DDA Casyopée. Their constructionist culture made the Greek researchers particularly sensitive to the expressive power of digital artefacts, which they had theorized in terms of "half-baked microworld", that is to say "pieces of software explicitly designed so that their users would want to build on them, change them or decompose parts of them in order to construct an artifact for themselves or one designed for instrumentation by others" (Kynigos, 2007, p. 336). This obviously influenced their (local) integration of Rabardel's and Vérillon's ergonomic perspective into constructionism, leading them to emphasize the instrumentalization dimension of the process of instrumental genesis. Indeed, instrumentalization captures the vision that artefacts can be transformed by their users in ways anticipated by the designers, but also in ways that are not anticipated (catachresis phenomena). Instrumental genesis results from the dialectic interaction between instrumentalization and instrumentation processes, and a constructionist perspective leads to emphasize the role of instrumentalization in this dialectic interaction. In the design of Cruislet and MALT, and in the tasks they proposed to students, the ETL researchers were particularly sensitive to the need to support this instrumentalization dimension of instrumental geneses and the students' creativity associated with it. In line with the constructionist culture, programmable simulations using the Logo language, open and challenging tasks with strong potential for the students' expressiveness were essential tools for this. The contrast with the DDA Casyopée was clear. This DDA was very innovative in terms of dynamic connections between representations – for example, it offered advanced tools for the functional modelling of geometric situations involving covariation of lengths and areas –, but the focus was on institutional graphic and symbolic representations, and the support offered by the DDA for their treatment within a given semiotic register or conversion from one register to another one, in line with Duval's semiotics. In terms of design priorities, therefore, more emphasis was placed on supporting instrumentation processes than on supporting instrumentalization processes, and the learning potential offered by the emergence and progressive transformation of alternative representations to the institutional ones was not considered. Such comparisons sharpened our vision of instrumentalization, of the dialectical games between instrumentalization and instrumentation, and made clear the influence on our vision of the theories with which we had networked ergonomic perspectives.

The cross-case study regarding the DDA Cruislet, involving ETL and DIDIREM, was also particularly insightful. It highlighted the profound differences between scenarios based on the theory of didactic situations and constructionism, although both theories share socio-constructivist principles. The concept of fundamental situation, the aim of optimizing the didactic potential of situations through the selection and management of didactic variables, and the attention paid to the organization of the dual processes of devolution and institutionalization, are at the very heart of the theory of didactic situations and of didactic engineering designs based on this theory. Such constructs are alien to constructionist scenarios. In these scenarios, the



aim of ensuring that the tasks designed and their technological and social environment offer strong potential for supporting the students' expressiveness is more important than the aim of achieving a precise learning goal and optimizing the possible trajectories. The Cruislet software, which offers various possibilities for simulating airplane flights over a map of Greece, was perfectly adapted to a constructionist approach to learning. For the French teachers and researchers, experimenting with it was a real challenge, and ultimately only possible within a specific project device, less constrained than the usual lessons. However, this experience was also a source of questioning and enrichment for us, as shown in (Le Feuvre, Meyrier & Lagrange 2010). Today, the evolution of the concept of didactic engineering carried out by ATD through the concept of study and research path and the related methodological tools such as question-answer graphs (Barquero & Bosch, 2015), undoubtedly provides a more appropriate framework for exploiting the potential of such constructionist DDAs in the French didactic culture.

Beyond this particular case, the ReMath cross-experiments showed that the productive networking of theories does not obey the same constraints when theories are networked, a posteriori, to analyze and interpret collected data, and when they are networked to build a conceptual framework to support didactic design, be it the design of digital artefacts or the design of tasks and learning situations involving them. The experimentation of Cruislet mentioned above showed that the logics underlying constructionist design and TDS design were so far apart that it was very difficult, if not impossible, to coordinate them in a conceptual framework. Significant distance was also observed in the conceptual frameworks supporting the design of situations involving the DDA *Casyopée* by the French DIDIREM team and the Italian UNISI team from the University of Siena. The UNISI design was guided by the theory of semiotic mediation (Bartolini Bussi & Mariotti, 2008), while the DIDIREM design, once again, was guided by TDS and Duval's semiotics. In line with their theoretical background, the Italian researchers conceived *Casyopée* as an instrument of semiotic mediation, and, in their design, they carefully organized the transition from semiotic signs attached to the artefact to mathematical signs detached from the artefact. Much time was devoted to this transition, in collective discussions led by the teacher without access to *Casyopée*. For the French researchers, in line with the TDS, *Casyopée* was a central element of the a-didactic milieu. Their design, while paying attention to the connection to be made between paper-pencil and instrumented techniques, did not conceive this connection in terms of transition. This was clear in the institutionalization phase where both instrumented and non-instrumented forms of knowledge were combined. Conversely, the various cross-case studies carried out in ReMath have shown the potential offered by the combination of constructionism, TDS and the theory of semiotic mediation in the a posteriori analysis of the experiments (see, for example, Maracci et al., 2013). In summary, the a posteriori combination of complementary and even conflicting theoretical perspectives in the analysis and interpretation of data can be very productive, while the combination of theoretical perspectives that are too far apart can make a coherent design impossible. More compatibility is needed. This is consistent with the distinction made between the combining and coordinating strategies in the scale of networking strategies (see Section 2) and confirms the pertinence of this distinction. Design based on theoretical networking requires coordination, and not just combination, of theories or theoretical constructs.

#### 4. A SECOND CASE STUDY: THE DOCUMENTATIONAL APPROACH TO DIDACTICS

The documentational approach to didactics (hereafter DAD) was born in a course given at the French Summer school of didactics of mathematics in 2007 (Gueudet & Trouche, 2009). It was motivated by the significant changes in the documentational work of teachers brought about by technological evolution, the increasing number of digital and online resources available to and used by teachers, and the growing role of professional and social networks in their professional activity. Digital technologies are therefore at the heart of DAD. As Gueudet (2009) explains in her analysis of the development of DAD, its source was her and Trouche's earlier work, which was mainly concerned with digital technologies. For Trouche, it was the contribution to the development of IA and to the SFODEM project, "a project of in-service teacher training, mainly at distance, aiming to support the integration of ICT in the teachers' practices by a rich offer of resources on a platform" (Gueudet, 2019, p. 19). For Gueudet, it was her research on learning processes with online exercises, first at university level and then extended at all levels of schooling, in particular in connection with the association of teachers Sésamath, an association that has produced very much used online resources, since its creation more than 20 years ago (<https://www.sesamath.net>).

The development of DAD in just fifteen years is impressive, as is impressive the number of researchers from different cultures and backgrounds who have contributed to it, as shown in (Trouche, Gueudet & Pepin, 2019). Another interesting element for our reflection is that, from the beginning, DAD has combined different sources of inspiration: IA and it can be seen as an extension of IA to the documentational work of teachers, but also many others. For instance, Adler's conception that "resources for school mathematics extend beyond basic material and human resources to include a range of other human and material resources, as well as mathematical, cultural and social-resources" (Adler, 2000, p. 210) is explicitly used in the extended definition of resources proposed in the DAD. Remillard's participative approach to the use of curriculum resources (Remillard, 2005), or Wenger's community of practice approach (Wenger, 1998), to mention just but a few, have also been very influential.

Over the last decade, we can observe a progressive evolution of the connections, both on the side of the founders of DAD and on the side of its contributors and users. Analyzing the first book published on this approach (Gueudet & Trouche, 2010), we have shown in (Artigue, 2019) that Gueudet and Trouche mention many authors in the two chapters they co-author, but that few theoretical connections are really elaborated beyond the foundational one with IA. For example, we observe only one example of local integration beyond the already mentioned one with Adler's conception of resources. It occurs when the authors link teachers' systems of documents and systems of activities by introducing a categorization of teachers' activities explicitly inspired by the study moments of ATD (Chevallard 2002). The foundational connection with IA leads to the distinction between resource and document, analogous to the distinction between artefact and instrument. Indeed, a document is defined as a hybrid entity consisting of updated/recombined resources and of a scheme of utilization of these. This connection also leads to the concept of documentational genesis with the dual processes of instrumentation and instrumentalization, and the development of schemes.

An interesting point is the awareness of researchers, since this emerging state of DAD, of the need for specific methodologies that allow them to approach the teachers' documentational activity in-class and out-of-class, with a long-term follow-up, and a close involvement of the teacher in the collection of data leading

to a reflective stance. Very soon, this methodology was given a name: “reflective investigation” and a specific tool was associated with it: the SRRS (schematic representation of resource system), and both became emblematic of the research praxeologies based on DAD. In the chapters of the same book written by other researchers, one can observe many cross-references but, once again, rather few advanced forms of networking, which is not surprising in this nascent state of DAD. In most chapters, the main purpose seems to be to make another approach understandable, or to contrast it with DAD. Only one chapter really goes further and opens the way to local integration (Trgalová, 2010). As explained in (Artigue, 2019, p. 100), “the author connects DAD with the model of teacher activity developed by Margolinas (2002) and Balacheff’s cKç model of conceptions (Balacheff 1995), two constructions already connected by these two authors to analyze teachers’ didactic decisions in (Balacheff and Margolinas, 2005)”. It is interesting to note that, in this particular case, the data used comes from a doctoral thesis whose methodology has stimulated some form of documentational work, which certainly makes the networking more practicable.

As shown in (Artigue, 2019), the theoretical core of DAD research praxeologies stabilized quickly, which contributed to the coherent structuring of these praxeologies, together with the privileged role given to the methodology of reflective investigation and the SRRS tool. This did not prevent the diversification of research *problématiques*, to which the increasing cultural diversity of the researchers contributing to DAD certainly contributed. When preparing our plenary lecture at the Re(s)ources 2018 conference, we asked Luc Trouche for a selection of references to get an updated vision of the theoretical connections in DAD, and we received a list of theoretical crossings with one or two references for each. The list mentioned thirteen theories and theoretical approaches. Some of them were already present in the first book such as information and communication sciences, Remillard’s approach or communities of practice, but also were new such as cultural historical activity theory, the theory of social creativity, constructionism or the meta-didactical transposition approach. Even if it remains partial, the analysis of the connections carried out in (Artigue, 2019) shows a clear progression in their quality. The collaborative work that has been developed, evidenced by the fact that most publications are co-authored, has obviously made this progress possible. There is no doubt that DAD researchers have been able to create communication devices and forms of collaborative work that help to meet the challenges raised by the increasing diversity of theoretical connections involved in the development of the theory, and also by the linguistic diversity of their community as shown by the DAD-MULTILINGUAL project (<https://hal.science/DAD-MULTILINGUAL/>).

In terms of research praxeologies, DAD today unifies, at a regional level, a diversity of research praxeologies aimed at better understanding of the transformations of teachers’ activity induced by the use of digital technologies, and at developing innovative uses of the increasing diversity of these technologies. As already mentioned, DAD research has created a methodological tool, the methodology of reflective investigation. This methodology, which provides a study technique well adapted to the research questions addressed by DAD, has become emblematic of its research praxeologies, and has been progressively enriched as shown in (Trouche, Gueudet & Pepin, 2019). The theoretical block of DAD research praxeologies has also been progressively enriched in dialectic interaction with the enrichment of their praxis block. For example, the study of documentational genesis over longer periods of time than was initially the case has led to the concept of documentational trajectory, which in turn has led to new methodological and representational tools (see, for example, Rocha, 2018).

In this second case study, the combined use of the networking and praxeological lenses helps us to understand a dynamic particularly rich in theoretical connections that has developed over the last fifteen years, to understand why we observe such a theoretical diversity in it, and how this diversity has gradually been better managed through more advanced forms of networking, and also thanks to the unifying characteristics of the DAD research praxeologies.

## 5. CONCLUSION

In this paper, we have shared some elements of the knowledge we have gained through our experience of networking between theories, focusing on research on teaching and learning in digital environments and the ways in which these influence teaching and learning processes in mathematics. If we return to the question that has guided the reflection developed in this paper: “To what extent are we now better equipped to face the challenge of theoretical diversity in what regards technology-based mathematics teaching and learning?”, our reflection makes clear that conceptual and methodological tools have come a long way since the issues raised by theoretical diversity were seriously put on the agenda of the international mathematics education community at the beginning of this century. As a result, as we have tried to show, knowledge has advanced both in terms of understanding the diversity challenge and in terms of developing strategies to address it. All the forms of networking identified in the scale of networking strategies have been used productively, involving a number of theories with different state of development, aims and scope. The connections established and the knowledge built now form a solid background for supporting research in this area. Networking research has also shown the interest of developing specific research praxeologies. These allow us to examine our respective research practices, their theoretical frameworks and their results without denaturing the theories and theoretical constructs involved, in order to build significant connections, and make clear their potential and limitations, as has been done, for example, in ReMath and in (Bikner-Ahsbals & Prediger, 2014). More globally, our reflection confirms the interest of adopting a praxeological lens to approach the diversity challenge.

The theoretical landscape of research on technology-based teaching and learning is a dynamic one. On the one hand, the development of multi-touch mobile technologies, of virtual and augmented reality devices, is the source of new interesting theoretical connections, such as the connection between IA and radical embodied cognitive science approaches presented in (Shvarts et al., 2021). On the other hand, the massive move towards online and hybrid mathematics teaching provoked by the pandemic situation has abruptly placed technology-based teaching and learning at the center of the educational agenda. This situation, and the resulting increase in educational inequalities, is also a source of new theoretical needs and connections, as illustrated by Borba (2021), for example. In this contribution to the special call on mathematics education research in pandemic times launched by the *journal Educational Studies in Mathematics*, Borba indeed reworks the theoretical construct of humans-with-media that he introduced years ago to emphasize that the production of knowledge results from the collective agency of humans and media (Borba & Villareal, 2005) to take into account the impact and agency of non-living things such as COVID-19. He also suggests combining this theoretical perspective with that offered by critical mathematics education (Skovsmose,

1994) to address the crucial issue of educational inequalities exacerbated by the pandemic. These are only two examples but there is no doubt that networking needs will continue to grow in this area of research as well as in mathematics education more generally.

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