DEVELOPING PROBLEM POSING IN A MATHEMATICS CLASSROOM

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Abstract

This is an exploratory study of 18 grade 9 students working on two problem-posing tasks involving the quadratic function. There are a variety of problem-posing strategies used by students, including the use of associated sub-topics, using the quadratic formula as a guide, working backwards, and adopting a trial-and-error approach. The free-posing task seems to help students to bring out more variety of sub-topics that they can connect, perhaps reflecting some confidence for such type of task. This is less so in the semi-structured task. It also appears that the number of sub-topics used is not dependent on student achievement type. Some implications for teaching and for teachers are also discussed. Specifically in the context of differentiated instruction in a classroom, problem posing activities can be one strategy to engage students. The findings of this exploratory study have the potential to add to the body of local knowledge about how problem-posing instructions can be engendered in the classroom to bring about deeper classroom engagement in mathematics.

Keywords: Mathematical Problem Posing, Problem Solving, Polya, semi-structured, free-posing

INTRODUCTION

Since the publication of Polya's (1945) *How to solve it,* problem solving has received much attention. This was especially so in the 1980s when there was a world-wide push for problem solving. Polya's problem solving model was in particular well-known as it is especially easy to "carry in the head" (Toh et al., 2008a, 2008b). In particular, Polya's four-phase model of problem solving, the fourth stage of problem solving is *Look Back* phase. This phase addresses the common phenomenon in school mathematics classrooms that the problem-solving process often ceases when a solution is reached. The *Look Back* phase extends beyond checking the reasonableness or correctness of the solution obtained; it includes adaptation, extension and generalisation of the original problem. Thus, Toh et al. (2008a, 2008b) renamed the *Look Back* phase as the *Check and Expand* phase. This refers to the stage of making the arrived solution of a problem as a starting point for further mathematics exploration, and marks the beginning of problem posing.

In this paper, we adapt the definition of mathematical problem posing operationally as referring to the process of generation of a new problem or a question by learners based on the given situation (Mishra & Iyer, 2013). It includes the generation of new problems in a mathematical context or the re-formulation of existing ones (Silver, 1994). In a classroom milieu, many of the students' mathematics experiences are solution-

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driven. Problem posing as a classroom activity can therefore offer a platform for students to transcend the fixation on problem solving where thinking is chained by prior knowledge and by set ways of seeing things. Teachers can also learn how much their students understand a mathematical topic through the problems that their students pose, since problem posing cannot be done without a context.

If learning mathematics is taken to involve creating meaning, then the ability to pose problems is an essential skill for creating that meaning to the learner. Meaningful learning will have a place to support arousing students‘ interest in the subject. Such a problem-posing classroom approach would have its place in supporting classroom enactment. In particular, in the Singapore context, we believe that problem posing would be able to support the recently released The Singapore Teaching Practice framework (Ministry of Education (MoE), 2017) by addressing the aspect of "arousing interest" for classroom instruction.

By a search of existing mathematics literature, there are relatively few studies on mathematical problem posing, and even fewer studies explore promoting student problem posing in the mathematics classroom (Chua, 2011). The exploratory study reported in this study attempts to add to the knowledge in the Singapore context through addressing the following research questions (RQs):

RQ 1 - What are the characteristics of students' problem posing in the semi-structured and free-posing tasks?

RQ 2 - What are students' selection of mathematical domain knowledge in problem posing?

RQ 3 - How does problem-posing performance vary across achievement levels?

Such knowledge on the characteristics of the students' posed problems is important in understanding how teachers can promote problem posing in classroom instructions. In answering the research questions, the products of the problem posing and the views of the tasks by students are examined.

RESEARCH BACKGROUND

Mathematical problem posing is identified as an essential mathematical activity and a companion to mathematical problem solving (Kilpatrick, 1987). Various studies have pointed to the importance of students' mathematical problem posing. This is related to students' exploration in mathematics (Cai, 2003) and the teaching and learning of mathematics (Crespo, 2003). Bransford et al. (1996) noted that developing students' ability to formulate their own problems is important for developing the mathematical thinking needed to solve complex problems. It has long been acknowledged that problem posing is an important intellectual activity in scientific investigation (Cai et al., 2015a). Brown and Walter (1993) noted that problem-posing activities in the classroom helped in lessening mathematics anxiety, in explicating misconceptions and in fostering group learning. Ellerton et al. (2015) noted that given that problem posing is closely interwoven with real-life situations, it can be seen as a natural link between formal mathematics instructions, problem solving and the world outside the classroom. Researchers have also found links between doing problem-posing tasks and students' being able to make connections and make sense of mathematics (Carrillo & Cruz, 2015). Hansen and Hana (2015) also argued that problem posing can give students the much needed ownership of their learning environment, since it is a natural component of inquiry-orientation and is grounded in the "belief of giving priority to the question over the answer." Elsewhere, Cai et al. (2015b) found that problem-posing activities can promote students' conceptual understanding, foster their mathematical communication and capture their interest and curiosity.

In the Singapore Ministry of Education (MoE) Mathematics Syllabus (2012), students are encouraged to "connect ideas within mathematics" (p. 8). The Singapore Mathematics Curriculum Framework (MoE, 2012) also places importance on the need for thinking skills and metacognition in mathematical problem solving. Such skills can be developed in the students through engaging students in problem-posing activities since posing a problem requires them not just to have the necessary concepts but also the ability to link these coherently together to form a problem. Students' responses to problem-posing tasks could provide a window through which to view students' ability to make connections within mathematics and a mirror that reflects the content and the character of their school mathematics experience. The importance of problem posing as a mathematical activity that could promote engaged learning provides the main impetus to the present study.

Depending on the purposes of the study, there are different problem-posing tasks in mathematics problem-posing research literature, ranging from free situation, structured situation, and semi-structured tasks (Stoyanova & Ellerton, 1996).These different contexts may result in different types of responses. This exploratory study aims to develop a description of what and how students problem pose in response to a semi-structured task and a free-posing types, and the mathematical domain knowledge that students' selectas they formulate their problems. This local description, though highly contextual, can provide insights into the problem-posing processes and provide clues about how students can be supported to problem pose.

The exploratory study can contribute to the knowledge of how students, being novice problem posers, can be taught heuristics that will build up their problem-posing skills. This will be helpful for the professional development of practicing teachers as such newly acquired knowledge on problem-posing will be able to facilitate teachers to stretch students beyond problem-solving and generate their own problems, which can be perceived as a form of empowering student learning. Through our collective experience with the mathematics classrooms, many teachers might not feel comfortable about problem posing, hence they will unlikely involve students in problem posing.

METHODOLOGY

Tashakkori and Teddlie's (1998) definition of a mixed methods design is adopted for the present study. It combines the "qualitative and quantitative approaches into the research methodology of a single study or multi-phased study" (p. 17). Green et al. (2006) and Denscombe (2007) argued that such a design besides providing a means to compensate the strengths and weaknesses of both approaches also provides a more complete picture of the research problem, hence in answering our RQs. Onwuegbuzie and Teddlie (2003) contended that mixed methodology allows one to make representation by way of extracting adequate information from the underlying data. It could also help in legitimizing "the validity of data interpretation" (p. 353).

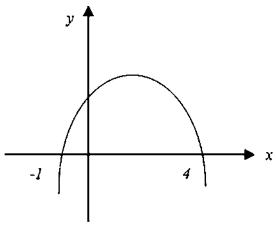
The problem-posing characteristics were inferred from the posed problems and the solutions. By engaging students to pose problems relating to a given stimulus and solve the problems they have posed, it is likely that the students would reflect within their posed problems and solutions, which is a crucial part of the learning experience which is an important part of the curriculum. This is supported by researchers such as Malara and Gherpelli (1994), who asserted that requiring students to solve their problems might prompt them to do deeper reflection on their own posed problems. This approach to engage the students to solve their own posed problems and to reflect on their problems is adopted by researchers on problem posing (e.g., Ellerton, 1986; Silver & Cai, 1996; Lowrie, 1999).

For practicing teachers, students' solutions are an indicator of how they have conceptualized their problems since the solving and posing processes are complementary (Contreras, 2007). Their problem-posing responses would therefore uncover the extent of their domain knowledge preference. Lowrie (1999) noted that the problem poser may not only focus on the underlying structures of the problem but also the extent to which the problem solver would be able to interpret the components of the problem. Students are therefore more involved in exploring the problems that they posed than when they were with the problems given by their teachers. The solution strategies and the use of the mathematical domain knowledge could give insights into the posed problems (Cai, 2003). Drawing from work by English and Halford (1995), problems were first classified as either solvable or non-solvable. A solvable problem has a well-defined initial state, a goal state and an inherent solution path.

The participants of this study were drawn from a sampling of a class of 18 Secondary 3 (grade 9) students from a secondary school, comprising 10 males and 8 females. The students were asked to complete two problem-posing tasks involving quadratic functions. The two problem-posing tasks were designed so that each could draw out a variety of problem-posing characteristics and to uncover the mathematical domain knowledge (sub-topics) used by the students.

Task 1: Free-posing Task

Write possible problems that are related to the quadratic curve given.



The quadratic curve for task 1 is commonly encountered by high school students in school algebra. In the syllabus documents, students are expected to be able to identify the geometrical properties and to solve problems involving the graph of a quadratic function. This free-posing task with a minimum context requires the problem poser to form the initial state, the goal state and the building of a context for the emerging problem. In this way, it attempts to uncover specific posing characteristics among the students.

Task 2: Semi-structured Task

The points (-2, 11) and (4, -1) lie on the curve given by the equation $y = ax^2 + bx + 3$.
Pose some problems arising from the problem stem.

Task 2 is more contextualized with elements of a given quadratic. The intention of the task is to investigate whether there are variations in posing strategies and in problem characterization in a more contextualized posing situation, with the given information about the two points and the form of the equation with a constant 3.

The students were given 20 minutes to complete each of task 1 and 2. After completing the problem-posing tasks, the students individually took a survey on their perceptions of the task difficulty, interest in the tasks, mental effort exerted and the extent in which they would like to attempt similar problem-posing tasks in the future. The students' performance in the two problem-posing tasks and the survey taken by them were used to answer RQ1 and RQ2.Three students were selected each from the different achievement levels (low, middle, high) with each student working on a new problem-posing task 3. A task-based interview was conducted with the three students. The interview provided a qualitative account of their problem-posing behaviour. The interview was audio-recorded and transcribed. The performance of this selected group of students on problem-posing task 3 and the task-based interview, together with the analysis of problem-posing performance across tasks 1 and 2 for the different achievement levels (low, middle, high) were used to answer RQ3.

Task 3:
Pose as many different problems where the solution involves:"the roots are $2 + \sqrt{3}$ and $2-\sqrt{3}$"

FINDINGS AND DISCUSSION

The categories of the posed problems with the exemplars are shown in Table 1. Across the two tasks, the common characteristics of the posed problems are the use of turning points (*maximum / minimum*), the *line of symmetry*, and the range of values for x. There were two non-solvable/vague problems:

- There is another line cutting through the curve. Name one curve. (free-posing task)
- If $x = 11$, or -1, solve the equation $y = ax^2 + bx + 3$ (semi-structured task)

These were not further analysed.

Table 1

Descriptions of posed problems

Category	Exemplar of students' response under this category.
Equation of graph	Find the equation of the graph
Maximum / minimum point	Find the maximum point
y-intercept	Find the point where the curve cuts the y-axis
line of symmetry	Find the line of symmetry.
Find *a, b* (in Semi-structured Task)	Solve for *a* and *b*.
Sketch	Sketch the curve $y = \frac{1}{2}x^2 - 3x + 3$
1-sub-topic	Find the coordinates of the curve at the y-intercept
2 sub-topics	Find the equation of the curve and the maximum point
Range of values for x	Find the range of values for x when y is positive.

CHARACTERISTICS OF STUDENTS' PROBLEM POSING IN SEMI-STRUCTURED AND FREE-POSING TASKS

One limitation to this exploratory study is that the some of the students' responses do not afford themselves for more in-depth analyses. Many of the students' solutions to their posed problems were too brief or left blank (44% in the semi-structured task, 50% in the free-posing task). The characteristics of the students' problem-posing for the two tasks are described based on the number of sub-topics, process of problem posing, and the degree of perceived difficulties. These are sufficient to describe the characteristics of the problem posing and also to shed light into some possible implications for practice and further research.

a) Number of sub-topics

The distribution of the number of sub-topics across the two tasks is shown in Table 2. For the free-posing task, majority of the students used two or more sub-topics, reflecting perhaps their confidence in bringing items together to form the problems. In the semi-structured task, more than half of the students used two or less sub-topics. Student G2 reflected that (s)he took on sub-topics in the formulation by looking at what were familiar:

"*I was using most of the questions given by the teachers in my exam paper itself. By seeing where I know that I've seen familiar equations and familiar answers by using the same thing of what they have given me*."

Table 2

Number of sub-topics

	Number of sub-topics (%)		
	1	2	3
Task 1 (Free posing task) (total 17)	11.8	64.7	23.5
Task 2 (Semi-structured) (total 13)	69.2	23.1	7.7

Note: Some students did not complete at least one of the two tasks.

b) Process of problem posing

Across the two tasks, students used various strategies in problem formulation. Students drew on their knowledge about the associated sub-topics, for example, *turning point*, and the *line of symmetry*, to formulate problems. The variety of sub-topics that were used perhaps pointed to their understanding of the mathematical structure which they had embedded in their posed problems.

The quadratic formula appeared to have "guided" students to the questions that they wanted to pose. Student H1 noted that the formula guided her/him to the question that (s)he wanted to ask:

"*I was trying to find numbers that can actually get to the answers because it was square root 3 and it was quite difficult because there was a divide by over 2A which was, … so I have to make the number divisible by 2*."

"*So before I begin, I actually tried thinking of numbers that can actually put into this formula so that I* *can actually get to the answer. Like I have to find numbers that can actually be divisible by 2 and getting a number that can actually get to square root 3. So I had to, so that the number can be square rooted …*"

Some students also appeared to fit in the values to arrive at the final answer through a process of working backwards. For example, student H2 noted that:

"*… so I can check what is A, B, C by working backwards or I can write the equation first. I will first solve the equation by trying to work backwards so I will try to solve the equation by coming up with random numbers. Or I can use my calculator. ….*"

Student G3 also reflected that:

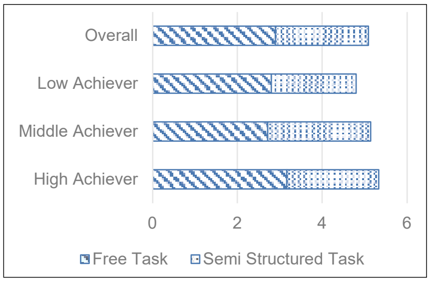
"*... when you pose a problem, you have to work backwards and create some numbers so that the questions can be solved. But as for problem solving, it is quite simple as you can just use some, add numbers and divide. And you are working forward and not backwards*."

There was also evidence of the use of a "trial-and-error" approach to problem formulation. From student G2 noted that:

"*I would have tried to sub(*stitute*) or treated an equation first to be able to solve. If I won't be able to do it I will restart it and try a different number that I can ... By going with guess and check method*"

c) Degree of perceived difficulty

The free-posing task seemed to elicit better perception of ease of posing compared to the semi-structured task, specifically with the high achiever group as shown in Figure 1. Student H1 felt that the problem (s)he had posed was suitable for her/his friends to answer because "*the teacher has already taught in class so they can use this as a practice question, maybe a possible question that can come out in the exam paper*." Together with student G1's earlier reflection, " *.... I was using most of the questions given by the teachers in my exam paper..*" students appeared to have the strong consciousness of posed problems as being linked to assessment. This perhaps can be indicative of their classroom mathematics experiences as being solution-driven.



Perception of Task as "Easy"

	Free Task	Semi-Structured Task
Low Achiever	2.8	2
Middle Achiever	2.71	2.43
High Achiever	3.17	2.17
Overall	2.89	2.22

Perception scale: 1(strongly disagree) to 4 (strongly agree)

Figure 1. Students' perception of task as being "easy"

SELECTION OF MATHEMATICAL DOMAIN KNOWLEDGE (SUB-TOPICS)

The distribution of mathematics sub-topics are shown in Table 3 and Table 4. The use of turning points (*maximum /minimum*) were most common in the responses in the free-posing task, and least likely was the choice of using y-intercept. Finding *a, b* was most commonly seen in the semi-structured task, and least likely are the use of *line of symmetry* and *range of values*. These perhaps reflected students' familiarity with these sub-topics in their problem-solving experiences, and specifically with their idea about finding unknowns. No students extended their posed problems to involve topics in other strands of mathematics. Perhaps, this absence of connecting to other mathematical ideas could be because they were novice problem posers. That almost all the sub-topics revolved around quadratics could be due to the 'recency' effect as they were just taught quadratics in their lessons prior to their problem posing exercises.

Table 3

Number of sub-topics in Task 1 (free-posing task)

	Task 1 (free-posing), total 17				
Sub-topics	Equation of graph	Max / min pt	y -intercept	Line of symmetry	Range of values
Overall (%)	64.7	82.4	17.6	29.4	17.6

Note: Students can use more than one sub-topic in their posed problems.

Table 4

Number of sub-topics in Task 2 (semi-structured task)

	Task 2 (semi-structured), total 13				
Sub-topics	Find *a, b*	Max pt	Sketch	Line of symmetry	Range of values
Overall (%)	84.6	7.7	30.8	7.7	7.7

Note: Students can use more than one sub-topic in their posed problems.

For the free-posing task, the use of *Equation of graph* and the *line of symmetry* in their posed problems was moderately corrected, with $r(15) = -.60$, $p < .05$. Students who used the *line of symmetry* in their posed problem would likely use the notion of *sketch* in their semi-structured task, with $r(15) =.56$, $p < .05$.

PROBLEM-POSING PERFORMANCE ACROSS ACHIEVEMENT LEVELS

Performance is described in terms of number of mathematics sub-topics in this exploratory study. Achiever types and number of sub-topics used in the two tasks are shown in Table 5 and Table 6. Achievement level was found to be not significantly correlated to the choices of sub-topics across the two tasks.

Table 5

Achiever types and number of sub-topics in free-posing task

	Free-posing Task, total 17		
	1-topic	2-topic	3-topic
Low Achiever (%)	20	28.5	40
Middle Achiever (%)	20	71.4	0
High Achiever (%)	0	57.1	40
Overall (%)	11.8	64.7	23.5

Note: 1 student did not complete the task.

Table 6

Achiever types and number of sub-topics in semi-structured task

	Semi-structured Task, total 13		
	1-topic	2-topic	3-topic
Low Achiever (%)	60	0	0
Middle Achiever (%)	60	20	0
High Achiever (%)	60	40	33.3
Overall (%)	69.2	23.1	7.7

Note: Five students did not complete the task.

Majority of middle achievers and high achievers posed two-topic problems in free-posing task. Low achievers tend to produce one sub-topic problems in the semi-structured task. More middle achievers produce one sub-topic problems in the semi-structured task. The choices of sub-topics used by the different achievement levels across the two tasks are shown in Table 7 and Table 8. Across the achievement levels, most students use two sub-topics for the free-posing task, and fewer use one sub-topic. This may suggest students' greater confidence in problem posing in the free-posing task. The finding is also consistent with an earlier study (Chua & Wong, 2012), which showed that students tend to posed problems that involved more topics in a free-posing task. Majority of middle achievers and high achievers posed two sub-topic problems in free-posing task. Low achievers tend to produce one sub-topic problems in the semi-structured task.

Table 7

Achiever types and sub-topics in free-posing task 1

	Task 1 (Free-posing), total 17				
	Equation of graph	*Max / min pt*	*y-intercept*	*Line of symmetry*	*Range of Values*
Low Achiever (%)	80.0	80.0	40.0	0	20.0
Middle Achiever (%)	57.1	71.4	0	42.9	14.3
High Achiever (%)	60.0	100.0	20.0	40.0	20.0

Table 8

Achiever types and sub-topics in semi-structured task 2

	Task 2 (Semi-structured), total 13				
	Find a, b	*Max pt*	*Sketch*	*Line of symmetry*	*Range of Values*
Low Achiever (%)	60.0	0	0	0	0
Middle Achiever (%)	80.0	20.0	40.0	20.0	0
High Achiever (%)	80.0	0	40.0	0	20.0

There were also variations in their perceptions about the problem posing across achievement types as shown in Figure 2. High achievers appear to have more positive orientation towards problem posing, specifically in having interest and trying it in the future.

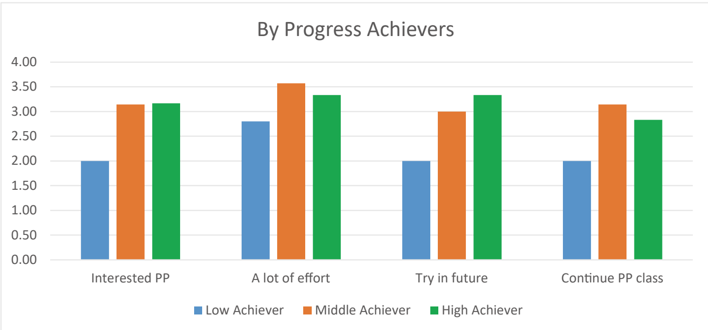


Figure 2. Students' perception of problem posing across achievement types

DISCUSSION AND IMPLICATIONS FOR TEACHING PROBLEM-POSING

Across the various problem-posing strategies used by students, namely, using of associated sub-topics, using the quadratic formula as a guide, working backwards, and adopting a trial-and-error approach, there appears that such an exercise is helping students to "think back" on the concept of quadratic function while posing. This may help students to in facilitating their recall of key mathematical concepts.Free-posing task seems to help students to bring out more variety of sub-topics that they can connect, perhaps reflecting some confidence for such type of task. This was less so in the semi-structured task. So a good starting point for teachers would be to introduce the free-posing type of tasks to students in the classroom.It appears that the number of sub-topics used is not dependent on student achievement type. In the context of practicing differentiated instruction in a classroom, problem posing can be one strategy to engage students in mathematical exploration

Student G2 noted that the whole problem-posing process itself can be "fun":

> "*I believe such activities are fun for the students because the students will know how much … be able to solve the question if they were to form their own personal question. If they have to think on how to solve it, which actually gives them the idea of what teacher have to think ... then they would be able to guess more on which equation to use for certain questions ...*"

There are also others who thought about the "utility" of problem posing in their classroom experience, as reflected by student G8:

> "*I learnt how to create questions and find this is quite useful as we have to do things reverse which helps me understand better. I've understood how to be a teacher too*."

Besides, both reflections of students G2 and G8 also point to the potential for problem-posing exercise as a way to engender interest and understanding in learning mathematics, for example, such activities

- as "*being fun for students ...*(G2),"
- bringing an understanding how the teacher poses questions (G8), and
- being able to bring about understanding because they "*have to do things reverse ...*(G8)"

These potential affective development that arise from problem-posing activities can bring about better student classroom engagement and may warrant further study beyond this exploratory study.

CONCLUSION

With the push for more subject discipline classroom engagement in the mathematics classrooms, there will be an increase in the emphasis in getting students to go beyond just problem solving and having the sense of inquiry in the real-world context which can be facilitated by problem posing. Given the importance of problem-posing as an emerging field of study in mathematics education research, the findings of this exploratory study have the potential to add to the body of local knowledge about how problem-posing instructions can be enacted in the classroom to bring about deeper classroom engagement in mathematics. Specifically, the study underlines the importance of planned approaches for the use of problem-posing activities in the classroom. Like problem-solving heuristics, teachers may have to teach problem posing explicitly as part of the classroom instructional programme. Students posing problems can bring about connections to real-life contexts, better student engagement, and may bring insights into students' understanding of mathematics. Specifically, problem-posing exercises can bring about greater awareness of the connections between the initial state and the goal state of a mathematics problem, and better cognizance of problem-solving strategies, mathematics content knowledge and processes.

Both problem solving and posing are influenced by affective factors, for example, beliefs, control, etc., and by the regulation of cognition. (English & Halford, 1995, p. 261). Besides this regulation of cognition, Schoenfeld (1985) also argued cogently about the importance of beliefs in solving. According to his model, beliefs referred to "one's mathematics world view, the set of (not necessarily conscious) determinants of an individual behaviour and about self, environment, topic and about mathematics" (p. 15). An area that warrants further study will be the effects of students' beliefs on problem-posing, their motivation and their background experiences in shaping how they construct problems. The poser, for example, had to be aware of when it was suitable to access a known problem and had to decide on the extent of correspondence between the two problem structures so as to determine the extent of procedural adaptation needed. Such a decision drew upon the poser's beliefs and the metacognitive control over the content knowledge.

A suggestion for a research agenda would be to further examine problem-posing responses across task types and involving other domains in school mathematics like statistics or probability. Such work could be useful in the design of intervention studies to promote problem-posing skills. More work may be needed to examine gender and problem-posing performance. There is still the need for more follow-up studies on a bigger sample. The present study nevertheless contributed to knowledge in this area.

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