FIFTH GRADE CHINESE AND U.S. STUDENTS' DIVISION PROBLEM POSING: A SMALL-SCALE STUDY

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Abstract

This study reports a small-scale international comparative study investigating rural elementary students' mathematical thinking on division, through analyzing the similarities and differences between division problems posed by elementary students in Inner Mongolia in China and Montana in the United States (U.S.). Bruner's (1985) paradigmatic and narrative modes of thought served as an analytic framework in this study. The primary data source for this study was students' responses to the open-ended prompt, "Write two different types of division problems." Each student's responses were coded according to the perspectives of paradigmatic and narrative modes of thought. The structures and contexts of posed problems and students' characterization of different division problems were examined. Our findings show that most students in both countries posed problems involving partitive (i.e., group size unknown) and equal groups division. No students in either country posed array/area problems. Of the ten common structures for division problems, students in China created problems aligned with six structures while the students from the United States used only two structures. An examination of the contexts used in each problem revealed that different types of food were the most common context used by students in both countries, although with unique cultural contexts. None of the students in either group situated their story problems in a rural context.

Keywords: problem posing; division problem

INTRODUCTION

This study analyzed the similarities and differences between division problems posed by upper elementary school students in the rurual, northern border regions of China and the United States (U.S.), Inner Mongoliaand Montana respectively.. This study investigated and compared their conceptual structures and contexts in the division that can be represented in the symbolic expression "a ÷ b = ?." In mathematics education, there is an increased emphasis on the development of a connection between conceptual and procedural forms of mathematics (Kobiela & Lehrer, 2015). Beyond a traditional emphasis on symbolic mathematical language, such a development requires the use of multiple representations to express thinking, make meanings, and demonstrate a deep understanding of the same mathematical procedure.

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Among various representations, writing has been recognized as a way to "boost learning in mathematics, develop mathematical understanding, change the pupil's attitude towards mathematics for the better, and help the teacher's evaluation" (Joutsenlahti & Kulju, 2017, p. 2). Problem posing, or using problem writing to represent symbolic mathematical problems, provides a unique window to develop a connection between conceptual and procedural forms of mathematics. It has also been recognized as an effective instructional strategy and assessment tool, mostly in mathematics and prose comprehension (Cai & Leikin, 2020; Mishra, & Iyer, 2015).

Previous studies (e.g., Cai, 1998; Cai & Hwang, 2002) explored how problem posing relates to problem solving and found a positive relationship between problem posing and problem-solving skills. Palmér and van Bommel (2020) recently showed that tasks posed by the children "shed light on their interpretation of what the original problem-solving task was really about" (p. 743). Problem posing has been recognized as a way to unfold new knowledge. Problem posing has beensuggested to be assessed to determine the extent to which creativity, including the constructs such as fluency, flexibility, and originality, is present (Leung & Silver, 1997; Shriki, 2013). Recently, research on how affective factors such as curiosity, interest, and enjoyment are associated with problem posing has increased attention (Cai & Leikin, 2020).

While problem posing can be used in many mathematical contexts and may have an impact on cognitive and affective domains, research has shown that posing problems for number sentences involving division is more challenging for students than number sentences involving other operations (English, 1997). Division is an indispensable arithmetical operation in the elementary school mathematics curriculum. It is at the uppermost level of elementary school mathematics operations. In China and the U.S., students are introduced to division in their third grade soon after they learn multiplication. Prior studies have identified many challenges that both students and teachers face in understanding the concept of division (Joutsenlahti & Kulju, 2017), especially while translating symbolic division problems into words (Ball, 1990; Jansen & Hohensee, 2016; Lo & Luo, 2012; Simon, 1993; Tirosh & Graeber, 1990). We are thus left with an incomplete account of children's understanding of division situations.

Several comparative studies (e.g., Cai,1998; Cai & Hwang, 2002; Ma, 1999) which involved Chinese and U.S. students can be found in mathematics education. However, as addressed by Wang and Lin (2005), cross-national comparisons are often ambiguous. In terms of sampling, for example, Chinese and U.S. students are often categorized as a homogeneous group without consideration of the similarities and differences among geographical locations within each country. Students in rural regions in China were also often excluded from these studies. With such lack of differentiation, cross-national comparisons may "mask underlying ethnic and cultural differences and thus prevent adequate interpretation of differences related to student performance" (Wang & Lin, 2005, p. 4). To develop a deeper and more discriminative understanding of how Chinese and U.S. students perform in mathematics, we need to consider whether sampling in the comparative study is comparable, and whether the comparative study targets and investigates specific topics or factors. In this study, we focused on the development of mathematical thinking and meanings on division problems among students in *comparable geographical regions* of China and the U.S. develop similar mathematical thinking and meanings regarding posing division problems.

THEORETICAL PERSPECTIVES

Two main theoretical perspectives guided the design of this study. They are (1) problem posing, and (2) division schema.

Problem Posing

There have been interests and efforts to incorporate problem posing into school mathematics (Cai & Leikin, 2020). Problem posing is "both the generation of new problems and the reformulation of given problems" and is considered to be a characteristic of creative activity or exceptional talent and a feature of inquiry-oriented instruction (Silver, 1994, p. 19). Stoyanova and Ellerton (1996) gave a framework that distinguishes forms of problem posing into the following three paths: (a) free problem posing, (b) semi-structured problem posing, and (c) structured problem posing. Free problem posing "provokes the activity of posing problems out of a given, naturalistic, or constructed situation without any restrictions" (Baumanns, & Rott, 223, p. 63). Although free problem posing is more demanding compared to the structured and semi-structured problem posing, it leads students to think independently and elicits authentic ideas. To optimize the exploration and analysis of students' thinking and ideas in problem posing, this study focused on free problem posing.

While students are given "the opportunity to construct their own representations of mathematical concepts, rules, and relationships" (Cai & Lester, 2008, p. 282), the variety of problem-posing types must be taken into consideration. In this study, Bruner's paradigmatic and narrative modes of thought served as an analytic framework to analyze the types of posted problems. A paradigmatic mode of thought is "context free and universal" (Bruner, 1985, p. 97). A narrative mode of thought focuses on "the broader and more inclusive question of the meaning of experience" (Bruner, 1985, p. 98). In relation to story problems, a paradigmatic mode of thought would require a focus on mathematical structures or models that are independent of a particular social context (Chapman, 2006). A narrative mode of thought in the context of story problems would require a focus on the social contexts such as the characters, objects, situations, actions, relationships, and/or intentions of the story problem (Chapman, 2006).

Division Schema

Piaget (1952) defined a schema as a conceptual representation of an associated set of perceptions, ideas, and/or actions. In Woolfolk's interpretation (1987), Piaget considered the schema to be the basic building block of thinking: a way of organizing knowledge. To describe an individual's schema in the meaning of arithmetic, Steffe and Cobb (1998) used Van Engen's (1949) operational theory of meaning that consists of three components (a referent, a symbol of the referent, and an individual) to interpret the symbol somehow referring to the referent. An individual's verbal interpretation, visual representation, or observable behavior can be taken as the referent for a symbolic operation and be considered as a demonstration of his/her arithmetic schema (Steffe & Cobb; 1998; Wilkins, Norton, & Steven, 2013). Steffe and Cobb (1998), for example, constructed the elementary division schema of a seven-year-old child through analyzing his verbal interpretations on whole-number symbolic operations like "24 ÷ 3." This study examined the division schema rooted in fifth-grade students' thinking through evaluating their verbal representations for symbolic division problems. Thus, story problems written for representing symbolic problems of division were examined.

From a paradigmatic perspective, most division story problems can be classified as either partitive (group size unknown) or quotitive (number of groups of unknown) division (Greer, 1992; NGA & CCSSO, 2010; Lo & Luo, 2012), and further classified into five sub-structures, as demonstrated in Table 1. This table was reorganized and revised based on NGA & CCSSO's (2010) "Common Multiplication and Division Situations" (p. 89) in the *Common Core State Standards for Mathematics* and Greer's (1992) summary table of "Situations Modeled by Multiplication and Division (p. 281). Some common types of division story problems cannot be classified into either *partitive* or *quotitive* structure. The *rectangular area* and *Cartesian product* types of problems are two types that cannot be classified.

(a) *Rectangular Area*: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?

(b) *Cartesian Product:* If there are 18 different routes from A to C via B, and 3 routes from A to B, how many routes are there from B to C?

Table 1

Common Structures of Division Story Problems

Structure	A: Partitive Division Group Size Unknown	B: Quotitive Division Number of Groups Unknown
1: Equal Groups	If 18 plums are shared equally into 3 bags, how many plums will be in each bag?	If 18 plums are to be packed 3 plums to a bag, how many bags are needed?
2: Part-Whole	A college passed the top 3/5 of its students in an exam. If 18 passed, how many students sat on the exam?	A college passed the 18 out of 30 students who sat on an exam. What fraction of the students passed?
3: Arrays	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will be there?
4: Comparison	A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A rubber band was 3 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
5: Rate	A boat moves 18 feet in 3 seconds. What is the average speed in feet per second?	How long does it take a boat to move 18 feet at a speed of 3 feet per second?

On the other hand, the classification of story problem contexts that deal with the narrative mode of knowing vary by the different interpretations of temporary situations and experiences.

METHOD

This study was designed to explore and compare the insights of division problem posing demonstrated by upper elementary students from China and the U.S. To this end, we examined the structures and contexts of division story problem posed by the targeted 5th grade students in each country. The following subsections detail the research method.

The theoretical and empirical literature discussed in earlier sections led to the following specific research questions:

1. To what extent can Chinese and U.S. students pose division story problems? What types of division problems do Chinese and U.S. students pose?
2. How similar or different do Chinese and U.S. students pose division story problems?

Participants

This study was an international comparative project investigating rural elementary students' mathematical thinking in the division. Purposive sampling (Leedy & Ormrod, 2005) was adopted to determine the subjects. The Chinese sample was from one typical public school in a big city in Inner Mongolia. The U.S. sample was from one typical public school in a small city in Montana. Although there is a significant difference in population size, both cities are located in rural, northern-border regions of each country with a sizeable minority population. Inner Mongolia is an autonomous region in China. Its two largest ethnic groups are Han (79%) and Mongol (17%). Montana has been home to seven federally recognized Indian reservations in the U.S. Its two largest ethnic groups are White (89%) and American Indian (6%).

Class sizes differ between the two countries, with about 58 students per class in China and 28 per class in the U.S. A total of 86 fifth-grade students (58 Chinese students and 28 U.S. students) participated in the study. Their homeroom teachers, who are mathematics teachers, administered the instrument to their students. No intervention was conducted in this study.

Instrument and Data Collection

A written instrument was chosen as the data collection tool for this study. The first three authors drafted and discussed the prompts assessing student division thinking. Both participating mathematics teachers from each country also reviewed the instrument to establish the face validatity of the instrument. Since this instrument is used in two different languages, we tried to ensure the two language versions are equitable. For example, we noticed that story problems in the U.S. are called "应用題" (application problems) in China. Therefore, we used the Chinese character "应用題" to represent story problems in the Chinese version of the instrument. The instrument consists of three prompts: (1) How would you define division? (2) Write two different types of division story problems. (3) Explain why these two division problems are different. Since this study focused on problem posing, only the results from the second prompt, "Write two different types of division story problems," was explored. The main source of data for this study is students' open-ended responses to the second prompt.

Data Coding and Analysis

Each student's response to the second prompt was coded according to the perspectives of paradigmatic and narrative modes of thought. The technique of content analysis was utilized to identify specific characteristics of collected data. Coding the collected data into categories relevant to the research objectives is an essential procedure of content analysis (Gall et al., 1996). In order to explicitly identify each posed story problem's division structure, the common structures of division story problems shown in Table 1 were used as a classification framework. Those collected story problems were classified into one of the following structures: *Partitive* division problems with the code "A," *quotitive* division problems with the code "B," other division problem structure such as *rectangular area* and *Cartesian product* problems with the code "O," and incorrect division problems with the code "NA." Each classified story problem was further labeled with one of the following categories of sub-structures: "1" for *equal groups*, "2" for *part/whole*, "3" for *arrays*, "4" for *comparison*, "5" for *rate*, "6" for *rectangle area*, "7" for *Cartesian product*, and "8" for any other exemplary story problem.

Different from paradigmatic types of structure, there is no prototype derived from the literature to serve as a classification framework for coding narrative context. This study adopted the method of open coding in which "the concepts emerge from the raw data and later grouped into conceptual categories" (Elo & Kyngas, 2008). Since the dividend is the quantity divided or grouped in a division problem, its context was coded to represent the narrative mode of thought in that problem. The open coding began with labeling the context of the dividend for each posed story problem. As open coding progressed, the codes were compared between problems to create concepts that further defined the categories.

The coding was developed through a series of stages. In the first stage, the first author coded the collected story problems using the classification framework for coding problem structures and the method of open coding for labeling and categorizing problem contexts. She examined each story problem and created a spreadsheet with a list of preliminary codes.

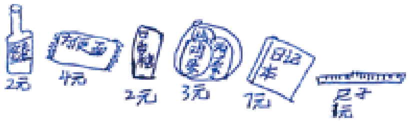
In the second stage, all of the authors later worked together to review, re-categorize, and adjust the preliminarily coded data. We tried to achieve coding consensus through this meeting, particularly on codes for those problem contexts that address the narrative mode of thought. The second author also paid particular attention to the equity issue of languages between English and Chinese. In the process, we also established a coding book.

In the third stage, the first author recorded the story problems using the established coding book and then compared her coded data with the coded data agreed or revised on the second stage. Any inconsistency was closely reviewed and discussed to ensure reliability before making the final decision. After completing the third step, we might still adjust based on a continued review of the coded data and literature. To accommodate the openness and variation of contexts, it is worth noting that a new contextual category might be added to the list of classification codes if necessary.

Table 2 provides the sample responses and their corresponding codes finalized after a series of coding stages. We analyzed the frequencies and percentages of division problem structures and contexts for each posed problem. The analysis results can allow us to know how students represent symbolic division problems in words and answer the research questions.

Table 2

Sample Responses and Their Corresponding Codes

ID	Story Problem	Structure	Context
C5.2	Xiao-Gang's mom and dad brought him to the beach to play. The temperature at the beach is 35.5 degrees Celsius. The temperature at Xiao-Gang's home is 20 degrees Celsius. What is the average temperature in degrees Celsius of these two locations? 	Partitive, Equal Groups	Measurement (Temperature)
C6.2	There are several products in Jia-Jia supermarket. Dad brought $60. What can he buy and how many can he buy?	Quotitive, Equal Groups	Money
U2.2	Joe has 40 M&M's and splits them between 4 friends. How many M&M's does each friend get?	Partitive, Equal Groups	Food (Candies)

Results

This section presents the results of the comparative analysis about the posed story problems, focusing on the problem structures and contexts along with descriptive statistics information. As shown in Table 3, among 116 story problems posed by 58 Chinese students, 95 (81.9%) are applicable division problems; among 56 story problems posed by 28 U.S. students, 48 (86.7%) are applicable division problems. Chinese students did not outperform their U.S. counterparts in posing appropriate division story problems.

Table 3

Distribution of Division Problem-Posing Performance across Chinese and U.S. Students

	China		U.S.	
Division Problems	Freq.	Perc.	Freq.	Perc.
Applicable Total	95	81.9%	48	86.7%
Not Applicable Total	21	18.1%	8	14.3%
Total	116	100%	56	100%

Problem Structures

In the study, except the story problems not written as an applicable division problem, all the posed story problem structures can be classified into one of ten common division structures shown in Table 3: (a) two primary division structures of *partitive* and *quotitive* and (b) five sub-structures of *equal groups*, *part-whole*, *arrays*, *comparison*, and *rate*.

As shown in Table 4, the story problems posed by Chinese students consist of six of the ten common division problem structures, while those posed by their U.S. counterparts only include two structures. The vast majority of the posed problems in both countries are A1 –partitive, equal groups division problems (China: 57.8%, U.S.:71.4%). Other applicable structures posed by students are A4 – partitive, comparison (China: 1.7%, U.S.: 0%), A5 – partitive, rate (China: 1.7%, U.S.: 0%), B1 – quotitive, equal groups (China: 15.5%, U.S.: 14.3%), and B4 – quotitive, comparison (China: 1.7%, U.S.: 0%) division problems. Students in China posed a more diverse range of problem structures than their U.S. counterparts. Additionally, no students in either country posed arrays problems. No *rectangular area*, *Cartesian product*, or other structures of division problems were generated by the participating students either.

Table 4

Distribution of Division Problem Structures across Chinese and U.S. Students

	China		U.S.		China		U.S.	
Structure	Freq.	Perc.	Freq.	Perc.	Freq.	Perc.	Freq.	Perc.
	A: Partitive Division				B: Quotitive Division			
1: Equal Groups	67	57.8%	40	71.4%	18	15.5%	8	14.3%
2: Part-Whole	0	0%	0	0%	2	1.7%	0	0%
3: Arrays	0	0%	0	0%	0	0%	0	0%
4: Comparison	2	1.7%	0	0%	4	3.4%	0	0%
5: Rate	2	1.7%	0	0%	0	0%	0	0%
Applicable	71	61.2%	40	71.4%	24	20.6%	8	14.3%

Problem Contexts

An examination of the context used in each problem revealed that different types of food were the most common context used by both groups of students. As shown in Table 5, nearly half (46.6%) of the story problems posed by Chinese students and over half (60.7%) of the problems posed by their U.S. counterparts used foods as the primary context. With such a significant number of the posed story problems written in the food context, it makes sense to classify this broad theme of food into smaller categories. This study refers to various food categories based on the elementary-aged children's consensus on distinguishing between fruit, snacks, candy, and foods served at meals. In Adams and Savage's (2017) study, food that children highly agreed upon as candy included mostly packaged food high in sugar and fat content, such as solid chocolate, lollipops, skittles; food they highly agreed upon as snacks included mostly non-perishable, often salty, convenience food, such as cereal, crackers, and pretzels. In Adams and Savage's study, children classified several "dessert-like food" such as cookies, ice cream, and cake-like foods less consistently as snacks or candy. In this study, we categorized those dessert-like food as a snack. Although most children in Adams and

Savage's study classified vegetables and fruits as snacks , this study classified these healthy and nutritional foods independent from snacks under a new subcategory. This study identified no context of meals or vegetables from the collected story problems. In summary, we broke down the broad category of food into three subcategories consisting of fruit, snacks, and candy.

A higher percentage (27.6%) of the story problems posed by the Chinese used fruit as context, specifically apples (21.6%), when compared to the percentage (10.7%) of those posed by their U.S. counterparts. Snacks, such as cakes, cookies, and chips, were the most frequently used food context by students in the U.S. (37.5%), but less by those in China (6.9%). Both groups of students posed similar percentages (China: 12.1%, U.S.: 12.5%) of their story problems in the context of candy. However, Chinese students only used the Chinese character "糖果"—a general word for candy in Chinese to represent candies, while their U.S. counterparts used more diverse terms, such as lemon drops and Starbursts.

Compared to the category of food, the rest of the context categories are smaller. The percentages of using daily necessities such as pens/pencils, books, and stamps between two groups of posed problems (China: 7.8%, U.S.: 7.1%) were closer than those in other categories. Chinese students were much more interested in splitting or grouping people than their U.S. counterparts (China: 11.2%, U.S.: 3.6%). Chinese students posed a higher percentage of monetary problems (6.9% versus 3.6%) than their U.S. counterparts. None of the U.S. students embedded the context of measurements, such as liters, temperature, and speed, into their problem posing as their Chinese counterparts did. None of the Chinese students embedded the context of toys into their problem posing as their U.S. counterparts did. The category of miscellaneous items consists of those contexts that occur only once.

Table 5

Distribution of Chinese and U.S. Students' Story Problem Contexts

Context	China		U.S.	
	Freq.	Perc.	Freq.	Perc.
Foods	54	46.6%	34	60.7%
Fruit	32	27.6%	6	10.7%
Snacks	8	6.9%	21	37.5%
Candy	14	12.1%	7	12.5%
Daily Necessities	9	7.8%	4	7.1%
People	13	11.2%	2	3.6%
Money	8	6.9%	2	3.6%
Measurement Attributes	8	6.9%	0	0%
Toys	0	0%	4	7.1%
Miscellaneous Items	5	4.3%	2	3.6%
Age	1	0.9%	0	0%
Balloons	1	0.9%	0	0%
Dog Treats	0	0%	1	0.9%
Fish	1	0.9%	0	0%
Pearls	1	0.9%	0	0%
Rocks	0	0%	1	0.9%
Terracotta Warriors	1	0.9%	0	0%
Applicable Total	95	81.9%	48	86.7%

Discussion

This study unfolds students' mathematical thinking from international perspectives through the lens of problem posing. Bruner's paradigmatic and narrative modes of thought were used as an analytic framework to examine the posted problems. As being conducted in two different rural regions, this study sheds light on the problem addressed in the introduction that other international comparative research simplifies countries into homogeneous entities.

The results provide the distribution of the posted problem types (structures and contexts). In this study, approximately 82% of the Chinese students and 86% of the U.S. students posed applicable division problems. The applicable problem-posing rates could confirm Cai's (1998) research results that both Chinese and U.S. students were able to formulate mathematical problems. Additionally, the results do not suggest that Chinese students outperformed their U.S. counterparts. On the other hand, different from Cai's study, this study found that Chinese students generated a more variety of problems than their U.S. counterparts. Thus, Chinese students' story problems are more diverse and inclusive than those posed by their U.S. counterparts. It is unclear whether or not the difference between studies was contributed by their distinction in research designs, such as sample sizes, research locations, and problem-posing tasks.

Both groups of students overwhelmingly posed the structure of *partitive* (group size unknown), *equal groups* division problems regarding division problem structures. The structure of *partitive, equal groups* division problems are often situated in an equal sharing scenario with a whole-number group as the divisor (Lo & Luo, 2012). Since the form of free problem posing was adopted in this study, its openness might make students intend to use a whole-number quantity as the divisor and consequently write more partitive, equal groups division problems. It is not intuitive to share a quantity among a fraction group such as the "1/4" group (Lo & Luo, 2012). It is unknown whether the structure of partitive, equal groups division problems would still be the most common problem among students when a divisor is a fraction group.

English (1998) and Lavy and Bershadsky (2003) stated that those problems without a clear mapping between the problem situation and the required operation are comparatively rarely posed. In English's study of addition and subtraction problems, such problems include comparison situations. Although this study focused on division operation, students also posed comparison division problems less than the equal-groups problems. Furthermore, none of the U.S. students posed a *comparison* problem. Given the limited range of problem types generated by the participating students from both countries, the question arises as to whether children could generate greater diversity in problem structures if their school experiences provide them more opportunities to consider multiple meanings of division. Thus, we wonder if some specific types of problems posing can be improved with some problem-posing instructions.

The contexts of the story problems collected in each country show the similarities and differences from narrative perspectives. Both groups of students overwhelmingly used food as context. Some contexts, such as Terracotta Warriors (China) and mooncakes (China) occasionally shown in those story problems, are indication of cultural contexts of the respective country. The contexts of rocks and dog treat in the U.S. might be related to the local biophysical environment or living habits. However, generally speaking, the inclusion of social-cultural contexts in the posed story problems was limited. For example, although the U.S. students lived in a spectacular valley adjacent to several stunning ski mountains and the well-known Yellowstone National Park, they did not incorporate any of those regional geographic features into their problem posing.

The results analyzed from the narrative mode of thought help reflect the true meaning and scope of the real-world contexts from the students' perspectives. We argued that students' problem posing might not be impacted by perfectly-phrased real-world contexts surrounding their communities. Instead, daily food such as fruit or various types of candy might better reflect their real-world contexts.

Several limitations exist while this study contributes to the ongoing efforts of understanding and explaining division problem posing. First, this study lacks sufficient information to generalize the posted problems in terms of learning settings and teaching approaches. This study did not collect information about the type of textbooks used by students and the type of story problems the students discussed or solved. This information could be relevant to the frequency of the type of problem around the examples provided by the students. In addition, this study did not investigate how teachers in both countries frame their teaching. The lack of investigation on teachers and their teaching approaches makes the contamination of teaching, a potential limitation regarding internal validity, unclear. If the teachers of the two participant groups joined in and finished an interview or a survey, we would know (1) the extent to which the teachers fostered the students' problem-posing activities in their teaching and (2) what contexts embedded the examples and problems they discussed in their classes. What teachers teach about division –for instance, types of division problems – would influence students' interpretation of division and writing of story problems.

Second, this study did not explore the relationship between problem contexts and structures. It is unknown whether using some contexts to pose a problem led to the way to pose a particular problem structure. It is unknown whether using some contexts to pose a problem led to the way to pose a particular problem structure. It is interesting to know, for example, whether the use of food contexts is associated with posing a partitive, equal groups, also called the equal sharing problem.

Third, this study did not relate the students' posted problems to their responses to the other two prompts: How would you define division? Explain why these two division problems are different. This study could characterize the students' problem posing more insightful by relating the types of posted problems to their responses to the other two prompts. It is interesting to know, for example, whether or not the problem contexts situated by the students would be less realistic if they defined division from a more procedural perspective, such as inverse multiplication.

Fourth, this study did not examine the relationship between problem-posing and problem-solving developments. It is unknown whether the problem contexts in which students situated their posed problems show consistency with the data involved and possible solutions. It is also unknown the complexity of numbers and steps to solve the posed problems. It is also unknown whether Chinese school students in this study were more likely to pose multiple-step or challenging story problems than their U.S. counterparts through more data analysis. Compared with U.S. students, Chinese school students have been more likely to choose complex tasks (Wang & Lin, 2005).

Lastly, this study did not provide students with sufficient opportunities to demonstrate their cognitive schemes. Since the students were only asked to pose two rather than as many different problems, the posted problems might only tell us the richness of problem types in their initial minds. In particular, this study collected and analyzed only a limited number of story problems from students in China and the U.S. It is crucial to take caution With such a small-scale sample size while interpreting the findings and recommendations generated from this study.

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