HISTORICAL COMPARISON AND ANALYSIS OF PROBLEMS AND PROBLEM-POSING TASKS IN CHINESE SECONDARY SCHOOL MATHEMATICS TEXTBOOKS

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Abstract

In this paper, the problems and problem-posing tasks in six series of secondary school mathematics textbooks were analysed, the distribution of the number, location and types of problems from the perspective of historical comparison, as well as the types of problem-posing tasks and the distribution of problem-posing tasks across content areas were studied. It was found that although the number of problem-posing tasks has increased, the percentage is still quite small with a maximum of 0.4%, and the distribution of problem-posing tasks across content areas is uneven. It was found that a large number of problems had been included in the content text section since 1990s. The distribution of these problems across grade levels and content areas are well balanced, indicating that problem-guided learning has become a new feature of the textbooks. From the perspective of types, these problems provide rich mathematical learning opportunities for students to acquire knowledge ("knowing" and "understanding") and to go through the thinking process ("to abstract and generalize" "to explore and discover" "to reflect and summarize"). However, the distribution of each type of problems across different grades and content areas are both uneven.

Key words: Textbook, Secondary school, Problem, Problem-Posing

INTRODUCTION

Tao Xingzhi, a well-known educationalist in China, once said "the starting point of invention is to ask". American mathematician Halmos believed that "problem is the heart of mathematics". The importance of problem is self-evident. Teaching through questions, as one of the effective methods to improve students'mathematical ability, has a long history. How to pose problems to effectively promote students' learning has always been an important aspect of mathematics education research. As an important carrier of curriculum ideas and important materials used by teachers and students, how are problem-posing tasks included in textbooks? To this end, we collected the data of problems and problem-posing tasks in six series of secondary school (grades 7-12) mathematics textbooks. From the perspective of historical comparison, we studied the following two topics: one is about the changes in the number and the distribution of the problems across topics, and the analysis of these changes from the perspective of learning opportunities provided by the problems; the second is the change of problem-posing tasks.

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BACKGROUND

Problems and Mathematics Learning

The psychological explanation of "problem" is that people have a problem when they are faced with a task and have no direct means to complete it (Wang, S. & Wang, A., 1992). As for the connotation of mathematical problems, there is no relatively unanimous and generally accepted view at present, and their denotation is mostly determined by classification. The most common way to classify them is by dichotomy, such as closed or open problems, conventional and exploratory problems, etc. (Nie, 2001). Mathematical problems can direct students' attention to specific learning content and prompt them to think, to understand and apply mathematics actively. Problems with different cognitive requirements often bring different learning opportunities. Problems with high cognitive requirements require students to think in a connected and comprehensive way, and these problems can often provide students with more opportunity to understand mathematics deeply. Therefore, many mathematical education researchers also classified mathematical problems from the perspective of cognitive level. For example, Getzels (1975) divided mathematical problems into three types from the perspective of "existing problems" and "discovered problems". Both Type-Case 1 and Type-Case 2 problems are ready-made problems, the former mainly involves the process of memorizing and extracting knowledge, while the latter requires students to analyze and infer by themselves. Type-Case 3 problems are not ready-made, requiring learners to discover and create problems, and these problems themselves are learning objectives. Stein and Smith (1998) classified mathematical tasks as low level tasks and high level tasks according to their level of cognitive demand. The low-level mathematical tasks involve two categories of cognitive demands: memorization and procedures without connections. The high-level math tasks involve two categories of cognitive demands: procedures with connections and doing mathematics.

Problem Posing and Mathematics Learning

Taking problem posing as a basic characteristic of mathematical activities, some outstanding mathematicians and educators, such as Polya, have proposed that it should be an important aspect of mathematics education for students to put forward mathematical problems autonomously. However, it was only in the last thirty years that problem posing has been widely concerned by mathematical education researchers, the main reason being the requirement of "innovative talent education" (Xia, 2005). According to Silver (1994), problem posing refers to both the generation of new problems and the re-formulation of given problems, and posing can occur before, during, or after the solution of a problem. Brown and Walter (2005) believed that it is impossible to solve a new problem without first reconstructing the task by posing new problem in the very process of solving and one did not appreciate the significance of an alleged solution without generating and analyzing further problems or questions. Thus, problem posing is not only a means to solve mathematical problems, but also a relatively independent mathematical activity. Many studies have shown that problem posing can improve students' conceptual understanding, creativity and mathematical attitude. For example, Silver (1997) argued that inquiry-oriented mathematics instruction, which includes problem-solving and problem-posing tasks and activities, can assist students to develop more creative approaches to mathematics. Through the use of such tasks and activities, teachers can increase their students' capacity with respect to the core dimensions of creativity, namely, fluency, flexibility, and novelty.

Problems, Problem Posing and Mathematics Curricula

In the early 1950s, the syllabuses for secondary school mathematics have shown that, whether it is the problem related to mathematical concept or techniques (skillful calculation and drawing, operating by formula, using mathematical table), it is of great significance to learn mathematics well (Chinese Ministry of Education, 1954). Accordingly, selecting appropriate materials to compile mathematical problems as the carrier of mathematical learning activities has always been the key consideration in the compilation of secondary school mathematics textbooks in China, especially when compiling the problems in examples and exercises. Many questions in examples and exercises have become classic questions and are still used today. Teachers also used these classical problems to compile variant problems for teaching. This has become one of the characteristics of secondary school mathematics teaching in China (Zhang & Wang, 2015).

As for problem posing, it did not appear in the mathematics curriculum documents until the 1990s. In the Mathematics Teaching Syllabus for Full-time Ordinary High School, problem posing was mentioned for the first time in the explanation of the ability to solve practical problems: "the ability to solve practical problems refers to the ability to propose, analyze and solve mathematical problems with practical significance or in related disciplines, production and daily life" (Chinese Ministry of Education, 1996). In 2000, in the revised versions of the syllabus of compulsory education and high school education, it was mentioned in the explanation of the sense of innovation that people could discover and pose problems from the perspective of mathematics (Chinese Ministry of Education, 2000a, 2000b). In 2003, in the high school mathematics curriculum standard, for the first time, problem posing was independent of problem solving, and was officially included in the curriculum objective: "to enhance the ability to pose, analyze, and solve problems from mathematical perspectives" (Chinese Ministry of Education, 2003). The latest high school mathematics curriculum standard clearly listed the "four abilities" objectives of mathematics curriculum: the ability to find and pose problems from mathematical perspectives, and the ability to analyze and solve problems (Chinese Ministry of Education, 2018).

Since problem posing has become an independent and important ability in mathematics curriculum's objectives, it is important to have problem-posing activities in the curriculum materials that teachers regularly use. Yet, there is at present a lack of research that focuses on problem posing in the mathematics textbooks used by students and teachers. Cai and Jiang (2015) found that the Chinese primary mathematics textbooks did contain problem-posing tasks, but the percentage of such tasks in each of the textbook series they examined was quite low and the distribution of problem-posing tasks across different content areas and different grade levels in these textbook series was uneven. Then the natural questions that arises is : how has the inclusion of problem posing in curriculum standards impacted secondary school textbooks? Are there enough problem posing tasks in current secondary school mathematical curriculum materials to realize the goals in the curriculum standard? Given the variety of ways to engage students in one form or another of problem posing, how exactly do secondary school textbooks include problem posing? What kinds of choices have textbook writers and curriculum developers made in creating existing materials? This paper attempts to answer these questions through analyzing the data of problems and problem-posing tasks in secondary school mathematics textbooks in China.

METHODS

Selection of Textbook Series

Given that problem posing was first explicitly raised in the 1990s' curriculum documents, we selected six series of secondary school mathematics textbooks published by People's Education Press (PEP) in the 1990s and after 2000, including three series of junior high school (grades 7-9) textbooks (PEP, 1992, 2004a, 2012) and three series of high school (grades 10-12) textbooks (PEP, 1997, 2004b, 2019), here we marked each series of textbooks with the year of publication of the first volume. The problems and problem-posing tasks were compared and analyzed. In this way, the trend of the change and focus becomes visible. In particular, for problem posing, we can see both its germination and its development.

Task Analysis

We first studied all the problems in the six series of textbooks to identify those that were problem-posing tasks. The basic standards are as follows:

(1) Operational definition: the term *problems* refer to tasks requiring students to complete by certain means; *problem posing* refers to the task that requires students to generate new problems based either on a given situation or on a given mathematical expressions or diagram (Cai & Jiang, 2015).

(2) Scope: compulsory part, excluding selected part, comprehensive practice activities , chapter introduction and chapter summary.

(3) Counting rules: an example or exercise was counted as one problem, and the additional question matching an example was included in this example and not counted separately; each of the questions in the columns of "thinking", "observation", "exploration" and "induction" were counted as one question by column.

The research was divided into two studies. In study 1, based on the existing research on the comparison of problem-posing tasks in elementary mathematics textbook, the percentages of types of problem-posing tasks and the distribution of problem-posing tasks in different content areas were compared. In study 2, according to the position of the problems in the textbook, we divided the textbook into three sections: content text, examples and exercises, and counted the number of problems in each section. Then we found that the problems in the content text increased most. Thus we further analyzed the specific manifestations and key characteristics of the problems of the content text section in the latest two series of textbooks one by one. Referring to the existing studies on the classification of mathematical task (Stein & Smith,1998), we divided these problems into five categories according to the opportunities they can provided for students: knowing(k), understanding(u), to abstract and generalize(a), to explore and discover(e), to reflect and summarize (r). Based on this taxonomy, we compared the percentage distribution of the five types of problems in different grade levels and across different content areas.

As for the types of problems, we analyzed and classified them according to the specific performance of the problem (the type of learning opportunities it can provides for students) and the key characteristics (the type of knowledge needed to solve these problems). Current cognitive and constructivist views of learning emphasize what learners know and how they think about it. In view of this, in the analysis of each problem, we focused on whether the problem enables students to possess a piece of knowledge or to experience a thinking process. If it is the former, according to the cognitive level, it can be divided into two categories:

knowing and understanding; If it is the latter, according to the way of thinking, it can be divided into three categories: to abstract and generalize, to explore and discover, to reflect and summarize. Table 1 shows the specific manifestations and key characteristics of each of the five problem types, as well as the sample questions we chose from textbooks.

Table 1

Specific manifestations, key characteristics and sample questions of five types of problems

type	specific manifestations	key characteristics	sample questions
k	Memorizing (concepts, formulas, theorems, methods, etc.) (see sample questions 1);Imitating (algorithms, reasoning, etc.); Judging (a concept, formula, theorem, etc.) directly or intuitively; Analyzing details and supplementing key elements, special cases, etc. (see sample questions 2).	Single, isolated	1. Do you remember how to draw an image of a function by tracing points? 2. Is the angle between lines *a* and *b* related to the position of point *O*?
u	To illustrate (concept, principle, algebraic expressions, etc.) by example (see sample questions 3); Multiple representation (meaning) (words, symbols, images, tables; Translation and interpretation between vectors, trigonometry, algebra, and geometry (see sample questions 4); Establishing connections in form and structures between different knowledge; Integrated application of knowledge to solve problems (Using multiple knowledge, involving a variety of situations and transfer of methods, requiring multi-step logical reasoning, etc.)	Associated, integrated	3. Parallel lines are very common in our daily life. Can you give some other examples? 4. Example 5 gives a property of arithmetic sequence. Can you explain this property of arithmetic sequence from a geometric point of view with the help of figure 4.2-4?
a	Analyzing (or verifying) multiple examples to abstract essential features, and generalize general mathematical concepts (or mathematical facts or principles) (see sample questions 5, 6); By analogy with similar mathematical objects to abstract essential features, and generalize general mathematical concepts (mathematical facts or principles); To abstract and generalize mathematical structures (graphic structures, reasoning structures, etc.) from the specific to the general (see sample questions 7).	General, structural	5. Is 30 + (–20) equal to (–20)+30? Please find some more numbers to try. What can you conclude from the above calculations? 6. What are the common features of the functions in questions 1 to 4 above? Can you summarize the essential characteristics of a function? 7. By summing up the generalities of the reasoning processes described above, can you arrive at a general structure for such reasoning?
e	To seek or create an idea or method (including algorithm, reasoning method, statistical method) in a given situation; Constructing a strategy or solution to solve the problem in a given situation and execute it (see sample questions 8); To discover (or guess) relations, laws, or properties by induction (or observation, operation, analogy, comparison, etc.) or deduction (such as specialization, generalization, etc.) (see sample questions 9).	Unknown, extended	8. If you want to know the ratings of a certain TV program in your area, can you help design a sampling plan? Discuss with your classmates based on the actual situation in your area. 9. Draw a parallelogram by definition, and look at it to see what else relationship of it's sides of it except for the fact that the opposite sides are parallel. What's the relationship between its angles? Measure it. Does it match your guess?

type	specific manifestations	key charact-eristics	sample questions
r	To improve the understanding of knowledge and methods by reviewing, comparing and summarizing their characteristics and functions (see sample questions 10); To achieve knowledge systematization by comparing mathematical objects (definitions, formulas, etc.) learned in different stages; To summarize the experience of learning a certain kind of knowledge and reflect on one's own thinking process (for example, how to study mathematical objects, how to discover mathematical properties, etc.) (see sample questions 11).	Systematic, metacognitive	10. Write the equations and solve it. Does the result agree with your previous estimate? What new insights do you have about the application of equations to practical problems by solving this problem? 11. Based on your previous experience in studying functions, how do you think we should study these functions?

It should be noted that these five categories are only a division based on the analysis of problems in the content text section, they cannot cover all types of problems. They are not and should not be separated from each other, and they may even have non-empty intersections. Although there is overlap between the different types, each category has its own focus that sets it apart from the others. For example, when a question focuses on asking students to explore and discover unknown knowledge or methods, abstraction or generalization is also indispensable in many cases. However, the problem of "to abstract and generalize" is more focused on drawing general conclusions. As an illustration, consider the question: "Take a number of different values of base a ($a>0$, and $a\neq1$) and draw the graph of the corresponding exponential function $y=a^x$ in the same plane rectangular coordinate system. Look at the locations, common points, and trends of these graphs. what do they have in common? Can you generalize from this the domain, range, and monotonicity of the exponential function $y=a^x$ ($a>0$, and $a\neq1$)?". In this question, although it is necessary to abstract and generalize the common characteristic of different functions, its main focus is to engage students to choose specific examples and drawing methods by themselves, and find specific conclusions through observation and comparison. Therefore, this task was classified as the type "to explore and discover".

To ensure the inter-rater reliability for the coding of the types of problems, a total of 104 problems, including 47 problems in junior high school textbook series (PEP, 2012) and 57 problems in high school textbook series (PEP, 2019) were randomly selected and coded by two researchers. We reached 96% in agreement with respect to the types of problems.

RESULTS

Historical Comparison of Problems and Problem Posing

The total number of problems and problem-posing tasks in junior high school textbooks and high school are shown in Table 2 and Table 3. In both junior high school textbooks and high schools, the number of problem-posing tasks is minimal. Nevertheless, we further studied the distribution of problem-posing tasks in different content areas and the percentages of problem-posing tasks of different types. Overall, the percentages of problem-posing tasks are quite small with a maximum of 0.08% in the latest junior high school textbook series (PEP, 2012) and 0.4% in the latest high school textbook series (PEP, 2019). And the distribution of the few problem-posing tasks in the four content areas (Function, Algebra, Geometry, Probability and Statistics) is very uneven.

Table 2

Total number of problems and problem-posing tasks in junior high school textbooks

Year	Total number of problems	Total number of problem-posing tasks
1992	3402	1
2004	2505	2
2012	2485	2

Table 3

Total number of problems and problem-posing tasks in high school textbooks

Year	Total number of problems	Total number of problem-posing tasks
1997	2151	0
2004	2496	2
2019	2629	11

In both junior high school or high school, the number of problem-posing tasks in textbooks is very small. We classified the problem-posing tasks in the six textbook series by the content area in which they were situated: Function, Algebra, Geometry, Probability and Statistics. Consistent with the research of Cai and Jiang (2017), we divided the problem-posing tasks into four categories: (1) Posing a problem that matches the given relationship expression(equation or function analytic expression) or image. (2) Posing variations of a question with similar mathematical relationship or structure. (3) Posing additional questions based on the given information and a sample question. (4) Posing questions based on given information. We found that the problem-posing tasks in junior high school mathematics textbook series only appeared in the field of Algebra, focusing on integral expression, equations, inequalities and so on, and only two types are involved, namely type (1) and type (4). Table 4 shows the distribution of 11 problem-posing tasks in the latest high school textbook series (PEP, 2019), and Table 5 gives some specific examples of the 11 problem-posing tasks.

Table 4

Distribution of 11 problem-posing tasks in the latest high school textbook series

	Function	Algebra	Geometry	Probability and Statistics	Total
Type (1)	4	0	0	1	5/45%
Type (2)	1	0	0	0	1/9%
Type (3)	2	0	1	0	3/27%
Type (4)	0	0	0	2	2/18%
Total	7/64%	0	1/9%	3/27%	11/100%

Table 5

Some specific examples of the 11 problem-posing tasks

SN	Content area	Type	Examples
1	Function	(1)	Try to construct a problem situation in which the relationship of variables can be described by $y=x\ (10–x)$.
2	Function	(1)	As shown in the figure, the graph of function $y=f\ (x)$ is composed of curve segment *OA* and straight line segment *AB*. Please put forward a practical problem that coincides with the graph of function $y=f\ (x)$.
3	Function	(2)	By analogy with the above generalization, write down a generalized conclusion related to "the image of function $y=f(x)$ is symmetric with respect to the y axis if and only if the function $y=f(x)$ is an even function".
4	Function	(3)	If $\sin\beta + \cos\beta = \frac{1}{5}$, $\beta \in (0, \pi)$. (1) Evaluate the expression $\tan\beta$; (2) Can you construct more problems for evaluating an algebraic expression by yourself according to the given conditions?
5	Geometry	(3)	If $a \perp \boldsymbol{\alpha}$ and the straight line b outside of plane $\boldsymbol{\alpha}$ is perpendicular to the straight line a, what conclusion can you draw? Can you pose additional questions and find more conclusions by yourself?
6	Probability and Statistics	(1)	Can you construct a practical problem to explain the meaning of equation $C_n^k \cdot C_{n-k}^{m-k} = C_n^m \cdot C_m^k$?
7	Probability and Statistics	(4)	The student union of one school wants to investigate the opinions on the student activity plan for this semester. You volunteer as a researcher and plan to sample 10% students of the school. What problems might you encounter in the survey sample? What will these problems affect? How are you going to solve these problems?

It can be seen that 64% of the problem-posing tasks belong to the area of Function, and these tasks cover type (1), (2) and (3); problem-posing tasks in the field of Probability and Statistics occupy the second place, including types (1) and (4); problem-posing tasks in Geometry only covers type (3); there is no problem-posing tasks in Algebra. Although the percentages of problem-posing tasks in textbooks is very small, and the distribution of the few problem-posing tasks across different content areas is uneven, all the four types are involved. In the Function area with the largest proportion of problem posing tasks, one problem posing task is arranged in the example section of the latest high school textbook series: "*Try to construct a problem situation in which the relationship of variables can be described by y=x (10–x)*". In the solution of the question, the textbook gives the demonstration process of how to construct a problem situation, and similar questions are set for students to construct the problem situation in the exercises section. Accordingly, posing a problem that matches a given expression has become the main type of problem-posing task. For type (4) not found in Function, the seventh problem-posing task under Probability and Statistics in Table 5 falls into this category.

In general, the number of problem-posing tasks in textbooks is very small, even in the latest textbook

(0.4%). Moreover, the distribution of the problem-posing tasks across different content areas is also uneven: there are the most problem-posing tasks in the field of Algebra in junior high school textbook series, while there are the most problem-posing tasks in the field of Function in high school textbook series. As for the distribution of types of problem-posing tasks, the latest textbook series involves all four types, but the distribution is also uneven.

DISTRIBUTION OF THE PROBLEMS

Distribution of the Number of Problems in Different Sections.

As can be seen in Table 2, the number of problems in the three series of junior high school textbooks shows a decreasing trend. Compared with the textbook series published in 1990s (PEP, 1992), the number of problems in the textbook series published in 2010s (PEP, 2012) decreases by 27%. As can be seen from Table 3, the number of problems in the three series of high school textbooks shows an increasing trend. Compared with the textbook series published in 1990s (PEP, 1997), the number of problems in the textbook series published in 2010s (PEP, 2019) increases by 22%. In order to further see the position distribution of these changing problems in textbooks, we divided the textbook into three parts(content text section, example and exercise) according to the position of the problems in the textbook, and counted the number of questions in the three parts respectively, as shown in Table 6 and Table 7.

Table 6

Percentages of problems in different sections of junior high school textbooks

Year	Mainbody		Exercises
	Context text	Example	
1992	140/4%	508/15%	2754/81%
2004	648/26%	230/9%	1627/65%
2012	536/22%	250/10%	1699/68%

Table 7

Percentages of problems in different sections of high school textbooks

Year	Mainbody		Exercises
	Context text	Example	
1997	39/2%	379/18%	1733/81%
2004	527/21%	360/14%	1609/64%
2019	523/20%	372/14%	1734/66%

In terms of quantity, the number of exercises in junior high school textbooks after 2000 decreases by more than 1000, the number of sample questions decreases by more than 200, and the number of questions in content text section increases by about 400, so the overall trend of the number of problems in junior high school textbooks is reduced. Since 2000, the number of exercises and examples in high school textbooks has not changed much, and the number of problems in content text section has increased by nearly 500. Therefore, the overall trend of the number of problems in high school textbooks is increasing.

It can be seen that a common change in the number of questions in junior high school textbooks and high school textbooks is that a large number of questions have been added in the content text section. So what are the new problems? Considering the consistency of the compiling concepts of different versions of textbooks since 2000 (the textbook series of PEP, 2012 is the revised version of the textbook series of PEP, 2004a, textbook series of PEP, 2019 is the revised version of textbook series of PEP, 2004b), we performed a detailed analysis on each problem in the content text section of the latest junior high school textbook series and high school textbook series, and classified these problems into five types according to their specific manifestations and key characteristics showed in Table 1. The number of these five types of problems in different grades and content areas respectively was then counted.

Percentages of types of problems across grades.

Table 8 and Table 9 show the number and percentage of different types of problems in the content text section of each grade in junior high school and high school respectively. We found that the type "to explore and discover" accounted for the highest proportion with nearly 40% in junior high school textbook, followed by type "knowing" and "understanding", the sum of the three types accounted for about 85%. While in high school textbook, the percentage of type "to explore and discover" is almost as high as "understanding", they accounted for 70% in total. No matter in the latest junior high school textbook series (PEP, 2012) or high school textbook series (PEP, 2019), the proportion of the type "to reflect and summarize" is very low.

Table 8

Percentages of types of problems across grade levels in the latest junior high school textbook series

	k	u	a	e	r
Grade 7(214/40%)	67/13%	46/9%	24/4%	70/13%	7/1%
Grade 8(190/35%)	30/6%	54/10%	29/5%	72/13%	5/1%
Grade 9(132/25%)	26/5%	23/4%	13/2%	68/13%	2/0%
Total	123/23%	123/23%	66/12%	210/39%	14/3%

Table 9

Percentages of types of problems across grade levels in the latest high school textbook series

	k	u	a	e	r
Grade 10 (302/58%)	69/13%	79/15%	15/3%	122/23%	17/3%
Grade 11 (221/42%)	42/8%	98/19%	6/1%	61/12%	14/3%
Total	111/21%	177/34%	21/4%	183/35%	31/6%

Combined with the amount of class hours of each grade, it's about 2 questions per class hour in the content text section of the six grades. The most significant difference in junior high school textbook was the types "knowing" and "understanding". There are more "knowing" problems in Grade 7 than in Grade 8 and grade 9, and the "understanding" problems in Grade 8 are significantly more than those in the other two grades. In high school textbook, there are significant differences in the two types of "understanding" and "to explore and discover". The "understanding" problems in Grade 11 are significantly more than those in Grade 10, while the "to explore and discover" problems in Grade 10 are significantly more than those in Grade 11.

In general, the number of problems in the content text section of the six grades of secondary school mathematics textbooks is very balanced. This is an indication that the textbooks not only pay attention to impart students the mathematical knowledge, but also engage them to think mathematically. On imparting knowledge, the junior high school textbooks stress equally "knowing" and "understanding", while high school textbooks focus more on "understanding" of knowledge; Pertaining to thinking mathematically, both junior high school and high school textbooks emphasize the importance of engaging the students "to explore and discover". Moreover, according to the classification of Stein and Smith (1998), about 80% of the problems in the content text section are of high cognitive level, and more than 50% of the problems are for students to do mathematics.

Percentages of types of problems in different content areas.

Table 10 and Table 11 show the number and percentage of problems in content text section in different content areas. Combined with the class hours in each area, the number of questions in the four content areas is evenly distributed, with about 2 questions per class hour.

In junior high school, the five types are different in terms of field distribution; Problems of the type "knowing" are the most in the Algebra area and the least in the Function area; Problems of the type "understanding" are the most in the Function area and the least in the Algebra area; The type of "to explore and discover" is most in the fields of Geometry and Probability and Statistics; The type of "to abstract and generalize" and "to reflect and summarize" are most in the field of Algebra.

In high school textbooks, according to the type, the distribution of "knowing" type in the four fields is more balanced, and there are a little more problems in the field of Algebra; The distribution of "understanding" type is slightly different in the four fields, with the most in Probability and Statistics and the least in Function; Problems of "to abstract and generalize" type are more in the field of Function and Geometry; The distribution of "to explore and discover" type in the four fields is relatively balanced, and slightly less in the field of Geometry; The distribution of "to reflect and induce" type in the four fields is slightly different, with more problems in the field of Geometry and Function.

Table 10

Percentages of five types of problems in different content areas in the latest junior high school textbook series

Content area (class hours)	k	u	a	e	r	Total
Function (29)	6/1%	16/3%	8/1%	16/3%	1/0%	47/9%
Algebra (107)	54/10%	38/7%	39/7%	58/11%	10/2%	199/37%
Geometry (123)	56/10%	60/11%	19/4%	119/22%	3/1%	257/48%
Probability and Statistic (20)	7/1%	9/2%	0/0	17/3%	0/0	33/6%

Table 11

Percentages of five types of problems in different content areas in the latest high school textbook series

Content area (class hours)	k	u	a	e	r	Total
Function (81)	32/6%	47/9%	10/2%	63/12%	12/2%	164/31%
Algebra (59)	34/7%	47/9%	0/0	49/9%	4/0.7%	135/26%
Geometry (47)	23/4%	35/7%	8/2%	30/6%	11/2%	107/20%
Probability and Statistic (52)	20/4%	48/9%	3/0.5%	41/8%	4/0.7%	117/22%

In general, the distribution of content text problems in the four content areas is very balanced from the perspective of curriculum time, with about two problems per hour of instructional time.. The four content areas in the junior high school textbooks all favor "to explore and discover". In the high school textbooks, the problems in Algebra tend to let students to acquire knowledge, while the problems in Function area treat acquisition of knowledge and experiencing the thinking process equally.

CONCLUSION

According to the data in this paper, the number and position of problems in secondary school mathematics textbooks have changed significantly since 2000. Before 2000, the problems in secondary school mathematics textbooks were mainly questions presented as exercises; questions for students to practice, accounting for 81% of the total. Since 2000, the location of the problems has changed, with about a few hundred new questions appearing in the content text section.

As for these "new" problems, according to Table 1, the main function of these new problems serve to guide students' learning of mathematics. Textbooks not only focus on imparting students knowledge by means of "knowing" and "understanding" types of problems, but also engage the students to experience the process of mathematical thinking through "to abstract and generalize", "to explore and discover" and "to reflect and summarize" types of problems. In the six grades of secondary school, the problems of "knowing" and "understanding" account for about 50%, and the problems of "to abstract and generalize" "to explore and discover" and "to reflect and summarize" account for about 50%. The proportion of "knowing" type problems decreased slightly with increasing grade levels: the proportion of "understanding" type problems increased slightly, the proportion of "to abstract and generalize" type problems decreased slightly, the proportion of "to explore and discover" type problems remained basically the same, and the proportion of "to reflect and summarize" increased slightly. Secondly, we find that new problems are mainly of high cognitive level, and the types of problems are different according to the characteristics of knowledge. Although the new problems cover different cognitive levels, the proportion of high cognitive problems above the "understanding" level is as high as 80%. In particular, the type of "to explore and discover" is unanimously preferred by all knowledge fields. This also shows that textbooks focus on engaging students to "do mathematics". Therefore, "problem-guided learning" has become a new feature of secondary school mathematics textbooks in the 21st century.

As for problem posing, although the type of problem "to explore and discover", which occupied the highest proportion in secondary school textbooks, show students how to discover and put forward mathematical problems through inquiry to a certain extent, there are still few opportunities for students to discover and put forward a question by themselves. Even in the latest textbook series, only 0.08% and 0.4% are problem-posing tasks. This can be interpreted in two ways. The first is, compared with the long-standing concern about problems and problem solving in mathematics curriculum, it has only been more than 20 years since the emergence and independent existence of problem posing in curriculum standard documents, which may be the reason that problem posing has not been in a dominant position in textbooks. The second plausible reason could be seen from the perspective of textbook editors, although the added questions in the content text section focus on guiding students' learning, it is also a demonstration of "how to find and pose problems" for students. As mentioned in the "words of the editor in chief" of the second set of high school textbooks:"we will pose problems whenever we have the opportunity. We hope that after reading many questions, you can pose new questions even if you can't solve a problem" (PEP, 2004b). This may be the subjective reason why the percentage of problem posing tasks are very low, namely, the editors focused on demonstrating how to pose problems, while the awareness of letting students pose problems by themselves was still relatively weak, so problem posing did not become a feature of textbooks.

In this case, we believe that it only needs a little effort to change from the status quo of problem posing in textbooks to letting problem posing be a feature of textbooks. We suggest three ways to make this change: first, to increase the number of problem-posing tasks by modifying existing questions. This is relatively easy, for example, by deleting some existing information or adding statements like "can you ask similar solvable questions" or "can you use this information to ask other questions" to turn a problem into a problem-posing task. Second, by designing high-level problem-posing tasks to provide students with more active and in-depth learning opportunities. This point requires the efforts of the editor, such as the carefully constructing a situation or designing a problem-posing task, so that students can put forward a series of questions of different difficulty, etc. Thirdly, by systematically considering the distribution of problem-posing tasks in terms of types, fields and grades, the balanced and continuous distribution of problem-posing tasks in the whole set of textbooks can be realized.

References

Brown, S.I.& Walter, M.I. (2005). The art of problem posing (third edition). *Lawrence Erlbaum Associates, Inc.*

Cai, J. & Jiang, C. (2017). An analysis of problem-posing tasks in Chinese and US elementary mathematics textbooks. *International Journal of Science and Mathematics Education*, 15 (8), 1521-1540.

Chinese Ministry of Education (1954). *Zhongxue shuxue jiaoxue dagang(xiuding cao'an)* [Mathematics syllabus for high school (Revision of the draft version)]. Beijing, China: People's Education Press.

Chinese Ministry of Education (1996).*Quanrizhi Putong gaoji zhognxue shuxue jiaoxue dagang (gong shiyan yong)* [Mathematics syllabus for full-time ordinary high schools (Trial version)]. Beijing, China: People's Education Press.

Chinese Ministry of Education (2000a). *Jiunianzhi yiwu jiaoyu quanrizhi chuji zhongxue shuxue jiaoxue dagang (shiyong xiuding ban)* [Mathematics syllabus for full-time middle schools of nine-year compulsory education (Revision of the trial version)]. Beijing, China: People's Education Press.

Chinese Ministry of Education (2000b). *Quanrizhi putong gaoji zhongxue shuxue jiaoxue dagang (shiyan xiuding ban)* [Mathematics syllabus for full-time ordinary high schools (Revision of the trial version)]. Beijing, China: People's Education Press.

Chinese Ministry of Education (2003). *Putong gaozhong shuxue kecheng biaozhun (Shiyangao)* [Curriculum standards of high school mathematics (Trial version)]. Beijing, China: People's Education Press.

Chinese Ministry of Education (2012). *Yiwu jiaoyu shuxue kecheng biaozhun* [Mathematics curriculum standard of compulsory education (2011 version)]. Beijing, China: Beijing Normal University Press.

Chinese Ministry of Education (2018). *Putong gaozhong shuxue kecheng biaozhun(2017 nian ban)* [Curriculum standards for high school mathematics (2017 version)]. Beijing, China: People's Education Press.

Getzels, J. W. (1975). Problem-finding and the inventiveness of solutions. *The Journal of Creative Behavior*, 9(1), 12-18.

Nie, B. (2001). Approach the mathematical problems' constitution and problem posing. *Journal of Guizhou Normal University (Natural Science)*, 19(1), 60-63.

People's Education Press (PEP) (1992). *Jiunian yiwu jiaoyu sannianzhi chuji zhognxue jiaokeshu (daishu, jihe)* [Algebra and Geometry textbooks for junior high school]. Beijing, China: author.

People's Education Press (PEP) (1997). *Quanrizhi putong gaoji zhognxue jiaokeshu (shiyanben) shuxue* [Mathematics textbooks for high school]. Beijing, China: author.

People's Education Press (PEP) (2004a). *Yiwu jiaoyu kecheng biaozhun shiyan jiaokeshu shuxue* [Mathematics textbooks for junior high school]. Beijing, China: author.

People's Education Press (PEP) (2004b). *Putong gaozhong kecheng biaozhun shiyan jiaokeshu shuxue (edition A)* [Mathematics textbooks for high school]. Beijing, China: author.

People's Education Press (PEP) (2012). *Yiwu jiaoyu jiaokeshu shuxue* [Mathematics textbooks for junior high school]. Beijing, China: author.

People's Education Press (PEP) (2019). *Putong gaozhong jiaokeshu shuxue (edition A)* [Mathematics textbooks for high school]. Beijing, China: author.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, 14(1), 19-28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *Zentralblatt Für Didaktik Der Mathematik*, 29(3),5–80.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics teaching in the middle school*, *3*(4), 268-275.

Wang, S. & Wang, A. (1992). *Cognitive psychology*. Beijing Normal University.

Xia, X. (2005). Reviewing and reflecting of teaching study on posing mathematical problems at home and abroad. *Journal of mathematics education*, 14(3).

Zhang, J. & Wang, R. (2015). Ways and methods of using variant materials in Chinese mathematics textbook. *Journal of Mathematics(China)* , 54(10), 1-6.

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