A CHARACTERIZATION OF THE PROBLEM-SOLVING PROCESSES USED BY STUDENTS IN CLASSROOM: PROPOSITION OF A DESCRIPTIVE MODEL

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Abstract

My Ph.D.'s research (Favier, 2022) aims at characterizing the processes used by students when they solve problems in the ordinary context of the classroom, i.e., when the problem-solving session is led by the teacher. The chosen problems may require students to make trials and errors. I consider two levels of characterization: the outer structure (Lehmann et al., 2015) of problem solving processes in terms of timing and organizing of processes and the inner structure (Ibid) considering heuristics. In this paper, I focus on the outer structure of the processes. Embedded cameras installed on the students' heads were used to collect audio-visual data as close as possible to the students' work. The recorded work of 33 groups of two or three students (20 groups at the primary school and 13 at the secondary school level) for a total of 79 students are coded independently by a research assistant and by us using the framework for the analysis of videotaped problem-solving sessions by Schoenfeld (1985). It consists of cutting the students' work into macroscopic chunks called episodes. The analysis of these empirical data leads us to discuss and enrich the descriptive model of problem-solving processes proposed by Rott et al (2021) with an additional dimension that allows us to take into account the interactions between students and teacher. The use of this enriched model allows us to identify three problem solvers' profiles in terms of the process implemented. Some of these processes are linear like Pólya (1957) proposed in his model. Some others are cyclics like in Schoenfeld's model (1985). The large majority are not supported by these two very well-known models which shows the contingency of the student's work and, therefore, the limit of these two models (in particular Pólya's model) used to teach problem solving.

Keywords: Mathematical problem solving, Descriptive process model, trials and adjustments

INTRODUCTION

Problem solving in mathematics as a method for developing students' learning is put forward by different actors in the educational system, whether they are researchers or institutional leaders, in many countries and at different school levels (Dorier & Garcia, 2013). Houdement (2009) distinguishes two functions among the learning objectives of mathematics problems. On the one hand, there are problems that contribute to the construction of mathematical concepts. These problems best cater to learning *through* problem-solving. On the other hand, there are problems that form a part of the mathematician's activity such

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as searching or validating, which can be used for *learning about problem-solving*. In this paper, the second category is the focus of my study. More specifically, my research aims*[1] at characterizing the approaches used by students when they solve mathematical problems in classroom. In particular, we have chosen problems that may require students to make trials and adjustments (often called trial and error in the literature). The complexity of the problem-solving processes is taken into account at different levels of analysis. Lehman et al. (2015) discuss the inner structure and the external structure. In this paper, I focus on the external structure, i.e., the temporal organization of the process.

THEORETICAL BACKGROUND

In this section, the main models of problem solving found in the literature are discussed, followed by a discussion of the similarity and differences across these models. The first model to be discussed is Pólya (1957)'s model, which is seminal in the field of mathematical problem solving. Pólya models the problem solving process through the sequence of four successive phases as shown in Figure 1.

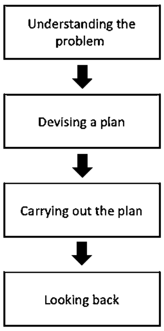

Figure 1. Pólya's model (1990, p. xxxvi)

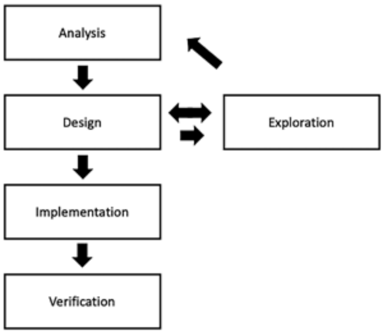

Figure 2. Schoenfeld's model (1985, p. 110)

To develop his model, Santos-Trigo (2014) explained that "Polya (1945) reflected on his own experience as a mathematician to write about the process involved and ways to be successful in problem-solving activities" (p. 498). This model has a prescriptive function since its intention was to teach students to solve mathematical problems. Moreover, this model has been used in many mathematics textbooks (Santos-Trigo, 2014; Wilson et al., 1993) and is still used as it is in French-speaking Switzerland (CIIP, 2018; CIIP, 2019).

The main criticism of this model is its linear nature. Nevertheless, it has served as a basis for several researchers to develop their own approaches, in particular Schoenfeld. Indeed, Schoenfeld (1985) has enriched Pólya's model by adding a phase that he calls *exploration* (Figure 2) to account for the part of the research that tends to move away from the understanding of the statement as such, which is therefore no

*[1] My doctoral research is part of a more global project, financed by the Swiss National Science Foundation, whose title is : « La résolution de problèmes comme objet ou moyen d'enseignement au cœur des apprentissages dans la classe de mathématiques : un point de vue fédérateur à partir d'études dans différents contextes ». Co-requestors Jean-Luc Dorier and Sylvie Coppé, Subside n°100019_173105 /1.

longer part of the appropriation phase, but which does not (yet) constitute a plan that needs to be implemented. By adding this phase, Schoenfeld differs from Pólya in that he affirms that the process is not necessarily linear, but can even be cyclical, that is, when there is a sequence between the three phases of analysis - planning/implementation - exploration in this order or if there is a back-and-forth between the planning/implementation and exploration phases. Mason et al (1982) also criticizes this linear aspect and propose a model which joins that of Schoenfeld on the cyclical character. Schoenfeld's model can also be described as normative, since it is supposed to represent "what may be called the *ideal* problem solver, or the most systematic behavior of good problem solvers" (Schoenfeld, 1985, p. 107). Wilson et al (1993) take the development of the model a step further by proposing a dynamic and cyclical interpretation of the different stages of Pólya.

Figure 3. Wilson et al 's model (1993)

Wilson et al's idea also breaks away from the idea of linearity, which is not very consistent with the genuine problem-solving activity (Ibid, p. 5). This model has also been used in a teaching perspective. Fernandez et al. (1994, p.196) noted that the model is being used as the framework in a mathematics problem-solving course at the University of Georgia to aid the discussion of issues involved with teaching mathematics problem-solving in elementary and secondary schools.

One of the common points of these three models is that they are built from the phases of the Pólya's model. This makes it possible to highlight their major differences, which concern the sequence possible of these phases. Another common point between the Pólya's and Schoenfeld's models is that they are built from the study of expert resolution. These processes are taken as a model in the sense of an example to follow. This is not the case with Wilson et al's model as these researchers attempt to take into consideration the fact that students may go back and forth between the different phases during their search which results in a more complex model. Despite this important difference, all three of these models are primarily used for educational purposes (Rott et al., 2021, p. 3).

Rott (2012) puts these different models to the test by analyzing the processes actually mobilized by 10-12 year old students to solve problems under laboratory conditions (students participated voluntarily and were taken out of school time). His analyses show that the processes mobilized by the students are not all linear (30 out of a total of 98 observations). This number, far from being negligible, highlights the fact that a strictly linear model such as Pólya's is not adapted to realistically describe students' work. Moreover, among these 30 resolutions, Rott counts 12 for which the process can be considered as cyclic and thus 18 of these resolutions present non-linear processes. Also, non-linear analysis models must allow for junctions between different phases of the resolution process. The cyclic Schoenfeld's model is a sub-case of all the cases that can be considered by a non-linear model. Furthermore, Rott observes that students do not necessarily make their solution plan explicit before implementing it, which leads him to group the planning and implementation phases together. Finally, students do not always move directly from ownership to planning and implementation, but often go through an exploration phase. This confirms, according to him, the importance of considering this exploration phase to account for the unstructured dimension of some of the processes.

Rott identifies some properties that a model should possess to describe problem-solving processes for 10-12 year-old students:

- There should be a distinction between structured and unstructured behavior (Planning and Exploration) as in Schoenfeld's model.
- It should be possible to intertwine Planning and Implementation.
- It should be able to display both linear and cyclic processes – with the majority of those processes being linear (Ibid, p. 105-106).

These different ideas are represented by the following diagram (Figure 4):

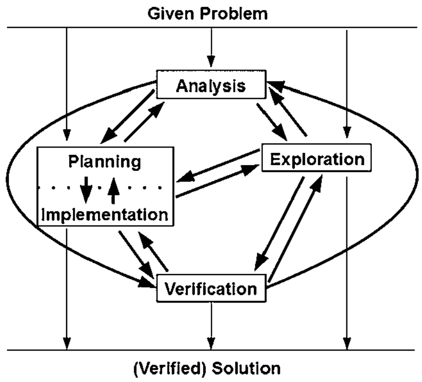

Figure 4. Rott's descriptive model of problem-solving processes (2012, 2021)

Thus, according to Rott, this model allows for a description of the problem-solving processes. A limitation of these different models is that they focus either on experts or on students who come to solve problems in the laboratory. None of these works studies the work of students in an ordinary classroom situation, with the possible interventions of a teacher. However, what interests us is precisely to characterize the approaches of students who solve problems during sessions lead by teachers. This leads us to formulate the following research questions:

- How can we characterize the problem-solving process of students when they solve problems in class?
- What characterizations emerge from this analysis?

METHODOLOGY

Research Context

My research was carried out in the canton of Geneva (Switzerland) with the intention of covering the three cycles of obligatory education. To do so, I focused on three different levels of education, from primary to secondary levels:

- Primary 4 (4P), which is the last year of cycle 1 and corresponds to students aged 7-8 years;
- 8th grade (8P), which is the last year of cycle 2 and corresponds to 11-12 years;
- 10th grade (10^{th}), which is the middle grade of the orientation cycle and corresponds to students aged 13-14 years.

Each degree is represented by two different classes.

The Problems

Problems that were used for this study can all be solved by trials and adjustments. The problems for the three different levels are presented below.

In 4P, Game Of Cards:

> Each card in my deck represents either a triangle or a square. I pull 15 cards at random. I count all the sides of the figures drawn on the cards I pulled and find 49. How many triangles and squares do you think I pulled?

In 8P, Dragons and company:

> On a computer screen parrots, crocodiles and dragons are seen. In total, I counted 20 heads, 72 legs and 30 wings. How many parrots, crocodiles and dragons are there?

In 10^{th}, Test ball:

> In order to renew its sports equipment, a school makes a first command of 2 rugby balls, 4 basketballs and 4 soccer balls for a total amount of 72 CHF. Then it makes a second order of 2 rugby balls and 2 basketballs and paid 30 CHF. We know that a rugby ball, a soccer ball and a basketball together cost 20 CHF.
> What is the price of each ball?

The 10th graders had learnt algebraic equations but had not learned to solve systems of equations. By making certain deductions, 8P and 10th graders could quickly find one of the answers. For example, in the Dragons and Company problem, the students could have used the data 30 wings to deduce that there are 15 animals that have wings (because dragons and parrots each have two wings). They then could deduce that there are 5 crocodiles. At this point, they could solve a problem of the same difficulty as the one proposed to the 4P students.

The Experimental Conditions

The research condition was not restrictive as it was intended to observe the students' approaches to solving problems as closely as possible to their usual working conditions. Thus, the arrangement that was proposed to the different teachers were the following:

- Introduction of the statement, answers to possible questions related to comprehension:
- 1st time of individual problem solving
- 2nd time of group problem solving

The teachers were asked to allow the students do as much problem solving as possible within the stipulated time limit of 45 minutes.

Data Collection

In order to attempt to document student's work, experimental data as close as possible to the students' work was collected. This being the case, the fact that my research was carried out in the usual classroom context, and that in order to guarantee the most ordinary and stable conditions for conducting the experiment, I faced certain constraints. In particular, the classic means of data collection (such as "thinking out loud" techniques, recordings with a camera on a stand, explanatory interviews (Vermersch, 1994)) were not feasible. Technical innovations, in particular the on-board camera, were exploited. According to Morieux (2016), the on-board camera allows "original shots and access to images of great didactic, pedagogical interest [...]" (p. 68). If using a camera external to the actor (classic data collection conditions) allows for a "third person" perception (Andrieu & Burel, 2014), filming with an on-board camera on the actor's body offers a "first person" perception (Ibid). This provides access to the students' workspace from their own perspective. In this way, the following became visible to us:

- anything the students pointed to or designated;
- items in students' field of vision;
- all the gestures that the students made in their field of vision;
- all manipulations performed with materials; and
- everything the students wrote, including those that were erased.

We also recorded everything the students uttered and probably (almost) everything they heard. These different elements are valuable for documenting the approaches used by the students during problem solving.

This first-person recording gave us the complete and real chronology of the different steps that constitute the students' problem solving. We should thus have access to the students' "private work" (Coppé, 1993), including all the trial-and-errors produced by the student and not only to the "public traces" (Ibid.) of their work that they want to show.

We therefore equipped one student per group with an on-board camera attached to the head. For each class, we collected the audio-visual recording of the work of each group filmed by the equipped student. It should be noted that we excluded from this corpus the few groups for which the audio or visuals were not usable.

Here is a summary of the number of groups filmed and the number of groups making up the corpus of data to be analyzed:

Table 1. Distribution of groups filmed and to be analyzed by class

Classroom	4PC	4PS	8PS	8PV	10CT	10LS	Total
Number of groups filmed	7	6	6	6	5	8	38
Number of groups to be analyzed	6	5	4	5	5	8	33

Each group is referenced by a code composed of the grade, the initial of the teacher (or the course of study for the 10th graders), the reference to the problem and the camera number. For example, the file 8PVDr7 corresponds to group 7 of Valerie's 8th grade class working on the problem Dragons and Company.

Framework For The Analysis Of The Empirical Data

To analyze the audio-visual data collected in each group, Schoenfeld's (1985) method of video analysis was used. It consists of parsing the students' problem-solving work into macroscopic chunks called episodes: "an episode is a period of time during which an individual or a problem-solving group is engaged in one large task or a closely related body of tasks in the service of the same goal." (Ibid, p. 292). Schoenfeld characterizes five types of episodes labeled by action verbs: read, analyze, explore, plan/implement, and verify. Here is a more precise description of how each of these episodes is characterized:

- *Reading*: this episode, as its name indicates, corresponds to the student's reading of the statement and also includes the time spent appropriating the various elements of the statement, whether in the form of silences, verbalizations, or silent re-reading.
- *Appropriation*: this episode corresponds to the attempts made to better understand the problem, to adopt a point of view and reformulate the problem in its terms, and to examine any principle that might be appropriate.
- *Exploration*: this episode differs from the previous one in its structure (it is much less structured) but also in its content, which moves away from the initial problem, in search of relevant information.
- *Planning/implementation*: in this episode, the emphasis is on the control dimension, which is why elements relating to the formation of the plan are not mentioned. Thus, it is more crucial whether the plan is structured or not, whether the implementation of the plan is methodical, whether the student monitors the process with feedback on the planning or on the evaluation at local or global levels.
- *Verification*: Schoenfeld considers the nature of this episode to be obvious.

He completes these five episodes with a so-called *transition* episode:

- *Transition*: this episode encodes the junctions between the other episodes, which are nodes during most of which decisions are made that can impact the solution in one way or another.

Rott (2011, 2012) adapted this method by introducing two additional episodes: *digression* and *writing*.

- *Digression*: this episode takes into account different student behaviors not related to the problem-solving task, such as when students talk about cartoon characters or TV series, etc., instead of working on the mathematical content.
- *Writing*: this episode allows for coding time spent writing (such as copying an answer) without getting new information or making real progress in problem solving.

Since this study took place in the classroom, with students working in groups, with the teacher ensuring the conditions of the study to be as close as possible to ordinary sessions. Therefore, the implementation of this method of analysis in the context of our research requires an additional adaptation. An additional category of episode, *regulation*, was introduced in order to take into account the exchanges that can take place between the students and the teacher.

Moreover, only one protocol per group was coded. In the event that two students produced different actions at the same time, the most informative interpretation from the point of view of the problem-solving was selected.

Finally, the audio-visual data were coded independently by a research assistant and by the researcher. We compared our coding results and when they did not coincide, we reached a consensus by recoding together. In terms of inter-coder agreement, we calculated a percent agreement*[2] (Jacobs et al., 2003, p. 100) of 0.76 for the nature of the episodes and 0.82 for the time codes.

RESULTS

The results of the parsing into episodes are presented in Table 2.

Table 2. Two examples of segmentation in episodes

	4PCJc7	
01:00	02:16	Reading
02:16	05:03	Implementation
05:03	06:28	Transition
06:28	08:46	Implementation
08:46	10:12	Regulation
10:12	11:53	Exploration
11:53	12:59	Transition
12:59	14:05	Implementation
14:05	14:15	Verification
14:15	15:15	Planification
15:15	16:23	Transition
16:23	16:55	Implementation
16:55	18:12	Digression
18:12	19:05	Implementation
19:05	19:22	Verification
19:22	20:15	Transition
20:15	23:39	Implementation
23:39	24:47	Regulation
24:47	25:32	Implementation
25:32	25:48	Verification

	4PCJc14	
00:20	00:30	Reading
00:30	01:42	Implementation
01:42	03:10	Reading
03:10	03:40	Implementation
03:40	05:07	Transition
05:07	06:40	Implementation
06:40	07:09	Regulation
07:09	10:40	Implementation
10:40	11:50	Transition
11:50	12:30	Implementation
12:30	13:03	Regulation
13:03	16:53	Planning - Implementation
16:53	17:58	Digression
17:58	20:09	Implementation
20:09	20:42	Planning - Implementation
20:42	22:54	Digression
22:54	25:45	Planning - Implementation
25:45	30:33	Digression
30:33	37:20	Regulation
37:20	43:20	Planning - Implementation
43:20	57:17	Digression
57:17	58:10	Implementation
58:10	01:00:08	Digression
01:00:08	01:01:40	Implementation
01:01:40	01:01:50	Verification
01:01:50	01:03:13	Transition
01:03:13	01:05:05	Regulation
01:05:05	01:06:14	Implementation
01:06:14	01:06:55	Transition
01:06:55	01:08:45	Regulation
01:08:45	01:10:20	Planning - Implementation
01:10:20	01:10:30	Verification

The first column corresponds to the start time of an episode while the second column marks the end time of this episode. The name of the coded episode appears in the third column. The tables obtained from the episode coding show a relatively high number of episodes and, moreover, a strong dispersion as shown

*[2] Calculations of the percentage of agreement could be calculated for the 8PV class, and the two 10th grade classes. For the classes of 4P and 8PS, we recorded the result of the confrontation on the same document as our initial coding. We thus lost the initial coding depriving us of the possibility to calculate the inter-coder agreement for these three groups.

by the box plot (Figure 5).

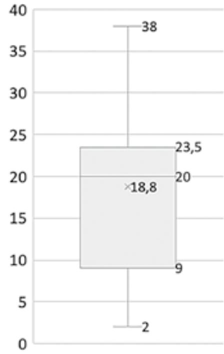

Figure 5. Box plot representing the distribution of the total number of episodes for each group

The students' work was divided into a minimum of 2 episodes and a maximum of 38 episodes. In addition, 75% of the segmentations have more than 9 episodes and 50% have more than 20. As it stands, this division and coding into episodes made comparisons difficult, since a great deal of richness was expressed through the chronology of the different episodes, their duration and their nature. In order to interpret these results, the complexity of these divisions was reduced by operationalizing the Rott's descriptive model (2012, 2021). Then, after having pointed out an important limitation highlighted by the experimental data, an enrichment of this model in order to allow the continuation of the analyses was proposed.

Using The Descriptive Model Of Problem Solving Processes

Following these authors, I call *phase* the different parts of this model; the term *episode* is reserved for the coding of audio-visual data. This model is completed by the number of passages from one *phase* to another which is inscribed in a circle next to the corresponding arrow. As far as our experimental data are concerned, this model seems to be operational for describing the work of groups of students for whom there was no teacher intervention, i.e., who did not present any regulation episodes. This concerns seven groups. The work of these seven groups using this model are presented below. Next to each number of passages (circled number), we have added another number, written with a grey italic font, which specifies the order of the passages in the model. Each of the following figures groups the results by proximity to known normative models.

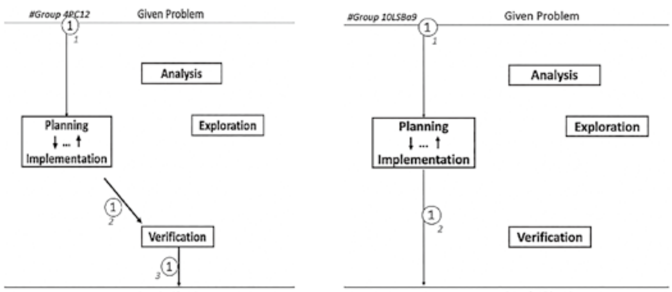

Figure 6. Groups whose resolution process is linear (in the sense of Pólya)

These first two groups show a linear "top-down" pathway of the model. This linear characterization corresponds to Pólya's model (1957). However, we can already see a small difference in that these two groups do not go through an analysis phase (i.e. *understanding the problem*) as envisaged by Pólya.

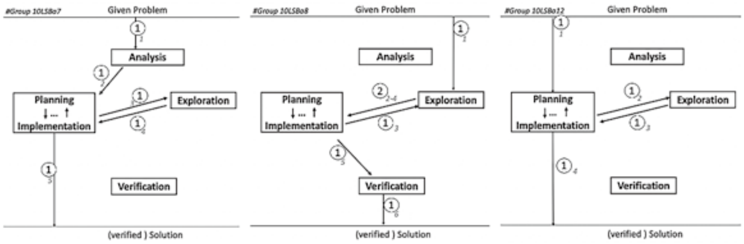

Figure 7. Groups whose resolution process is cyclic (in the sense of Schoenfeld)

These three groups are comparable in terms of the back and forth between the planning-implementation and exploration phases. This cyclical aspect of part of the process can be related to Schoenfeld's model (1985). As we noted a few lines above, these three groups illustrate three different ways of starting the resolution. Only group 10LSBa7 (on the left in Figure 7) is faithful to Schoenfeld's model since the process starts with an analysis phase. The other two groups highlight the need to enrich Schoenfeld's model by planning to enter directly into a planning or exploration phase as allowed by the model of Rott et al.

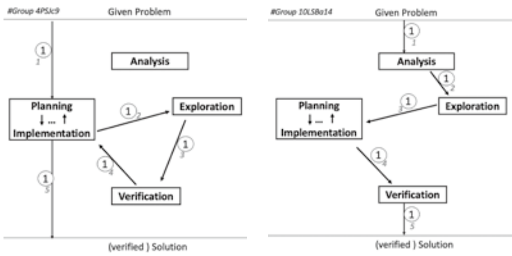

Figure 8. Groups whose process is not related to a normative model

Finally, these last two groups mark a break with the previous five groups. The one on the left (4PSJc9) shows an "ascent" in the model after the verification phase. The one on the right (10LSBa14) shows an appropriation phase followed by an exploration phase which corresponds to a sequence not supported by Schoenfeld's model. The processes of these two groups are not similar to the two models mentioned above. It allows us to understand and confirm (Rott et al., 2021) that the Pólya and Schoenfeld's models are not sufficient to account for the contingency observed in the classroom.

All of these seven groups highlight the operationality of Rott et al's model since it allows for the description of processes that are both linear, cyclical but also those that may escape them due to the more dynamic and non-linear aspect of the course. It also offers the possibility of describing processes that begin with a planning-implementation or exploration phase and not just an appropriation phase. This being said, the confrontation of this model with the other experimental data at our disposal, points to an important limitation which has led us to enrich it.

Proposal Of A Descriptive Model Of Problem-Solving Processes In The Case Of Teacher Interventions

To make this limitation explicit, we note that the problem solving by the subjects in the Rott et al. (2021) study was accompanied by as little help as possible from a tutor. It appears that this help was reduced enough to be neglected in the coding of the data. In this study, the teachers' management of the session and the interventions they might have had with certain groups of students were far from negligible. Indeed, to characterize the problem-solving process of the students in class requires the integration of the participation of the teacher, which necessarily brings a new complexity in the description of the phenomena. These intervention by the teachers were not taken into account by Rott et al. Moreover, the moment when the teachers intervened cannot be considered as being of the same nature as the other phases of the model. We therefore propose to enrich this model by adding a dimension to account for *regulation*. Thus, this additional dimension is on a different plane than resolution and we choose to materialize it in a third dimension (in the geometrical sense), in the enriched model. By testing this proposal for an enriched model on my experimental data, it appears that this *regulation* phase can be connected with all the other phases of the basic model. Double arrows are therefore necessary to represent these different possible connections. Moreover, we could observe some groups for which *regulation* occurred at the very beginning of the research, which explains the arrow that starts from the problem and points to regulation. In the same way, it happened that the solution was found or checked during a regulation: either because it was the teacher who did the work, or because he guided it. It is therefore necessary to add an arrow that starts from the *regulation*'s phase and points to the solution. Therefore, the model I use to describe the problem-solving processes in cases where the teacher intervenes is the following:

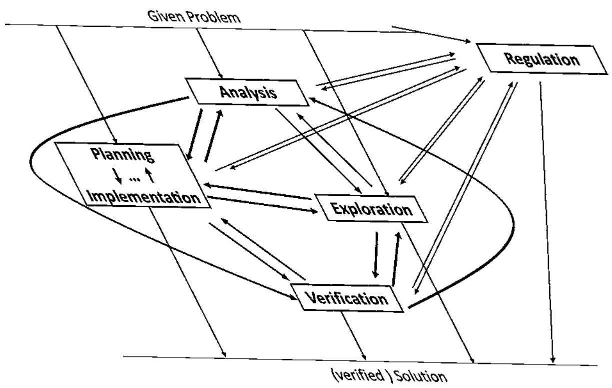

Figure 9. Proposed model to describe students' resolution processes in the case of teacher interventions

The following figures (Figure 10) show two examples that visualize the descriptions I obtain using this model:

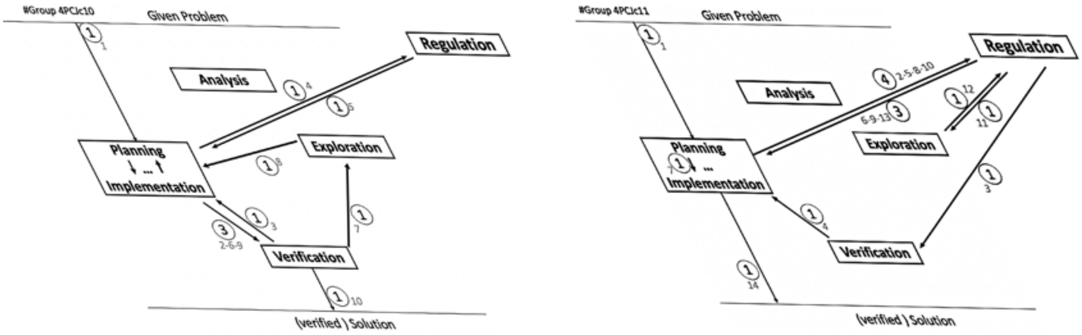

Figure 10. Two examples of students' resolution processes

The comparison of the different processes also reveals three general tendencies: linear processes (in the sense of Pólya), cyclical processes (in the sense of Schoenfeld) and those, very dynamic, which are not taken into account by these two models.

Linear processes

The three coded groups 8PSDr3, 8PVDr2, and 10CTBa12 show a linear process that is characterized by a solely top-down model path. The regulations do not deviate from the linear path of the model run.

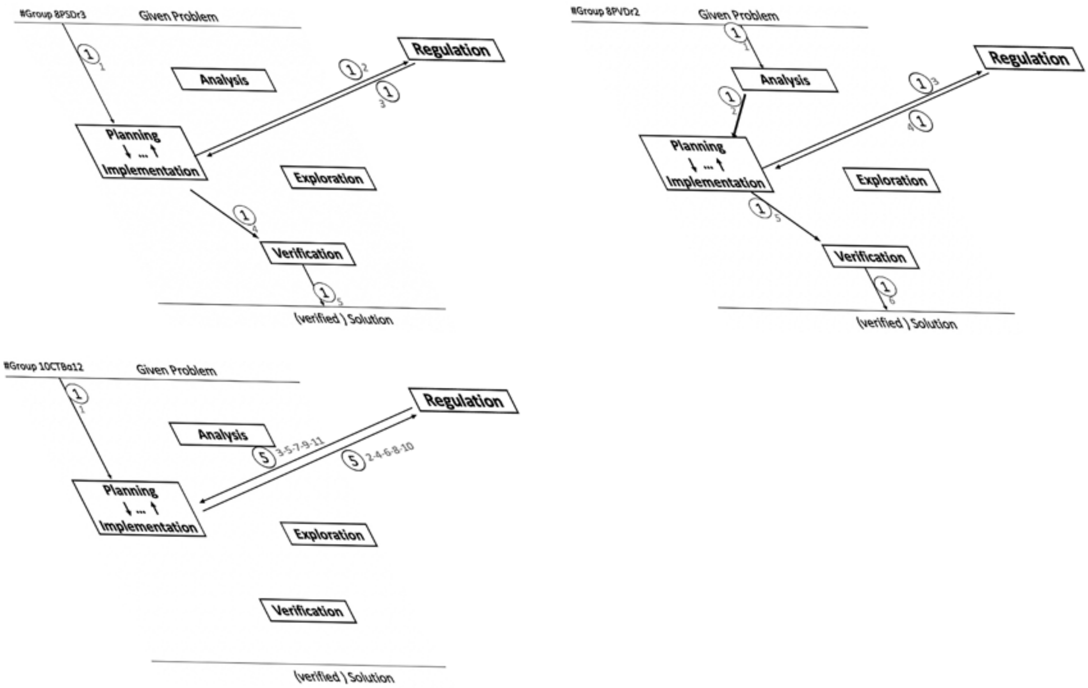

Figure 11. Groups whose process has a linear trend (in the sense of Pólya)

These graphs describe very different realities that are highlighted by the number of regulations (number circled next to the arrows that start or point to the regulation). Indeed, the group (10CTBa12) shows many regulations while the other two groups (8PSDr3 and 8PVDr2) have only one regulation each. I come back to this idea in a following section.

Cyclic processes (in the sense of schoenfeld)

A process to be cyclical (in Schoenfeld's sense) if there is a sequence between the three phases of analysis - planning/implementation - exploration in this order or if there is a back-and-forth between the planning/implementation and exploration phases. Thus, among the set of processes analyzed, the following three correspond to this cyclical characterization in Schoenfeld's sense:

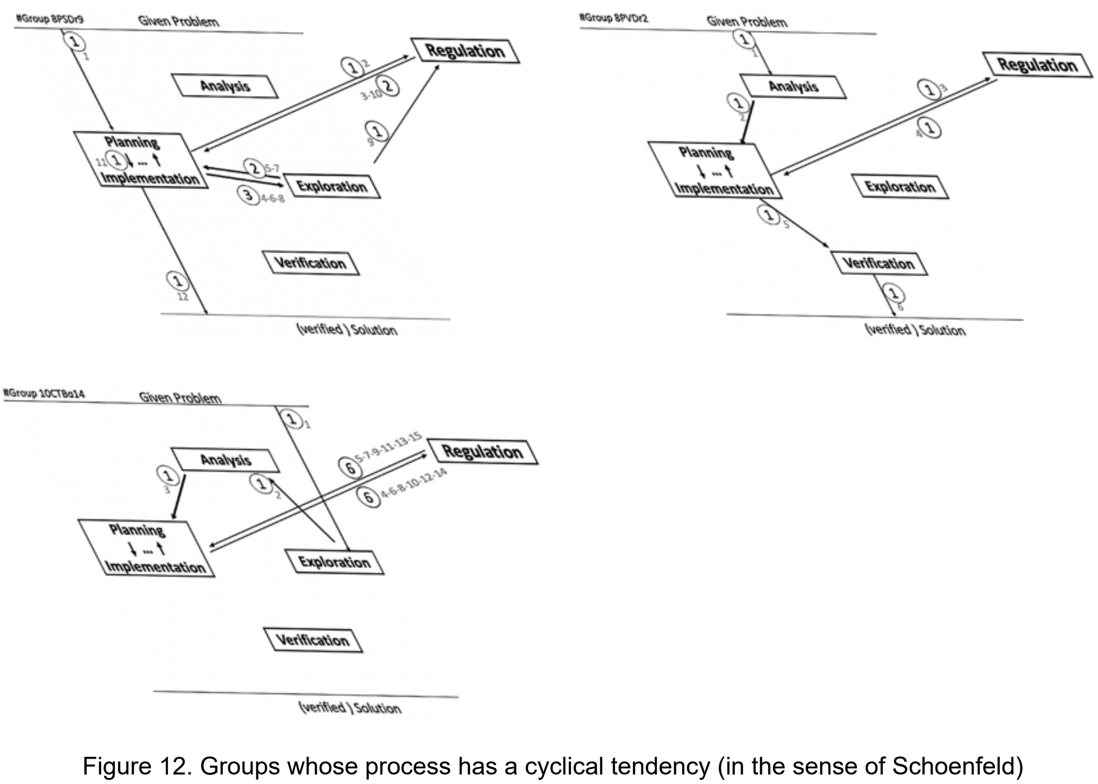

Figure 12. Groups whose process has a cyclical tendency (in the sense of Schoenfeld)

The first two groups show direct back and forth between the planning/implementation and exploration phases, corresponding to steps 4-5-6-7-8 for the 8PSDr9 group and steps 6-7-8 for the 10CTBa13 group. There is also some back-and-forth regulation: steps 9-10 for the 8PSDr9 group and steps 11-12 for the 10CTBa13 group.

Some groups can be associated with the Schoenfeld's model insofar as they are the regulations that allow for movements described by this model. Three groups are representative of this aspect as shown in Figure 13.

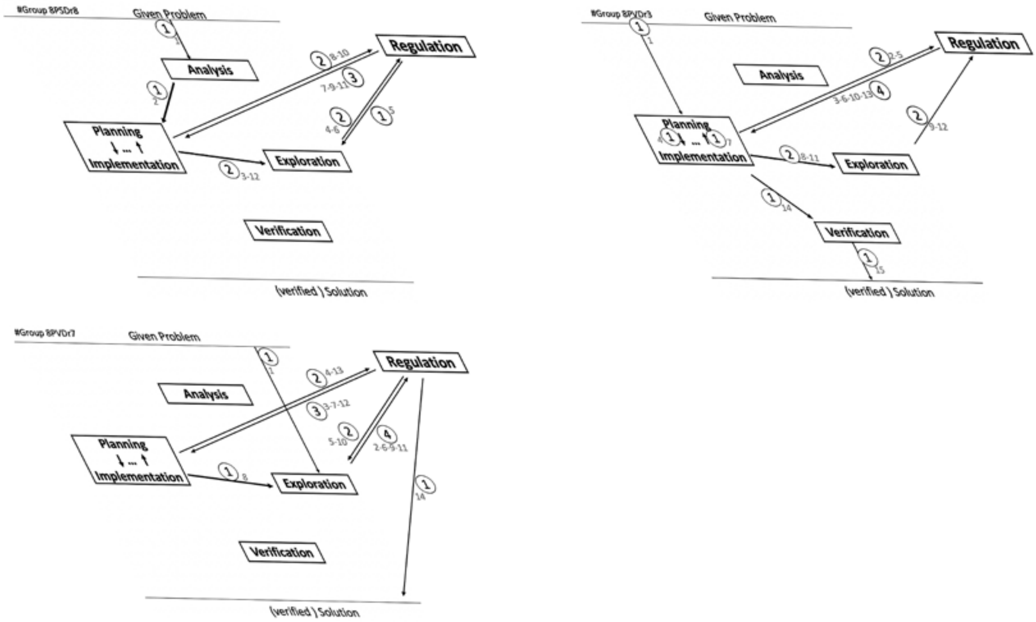

Figure 13. Groups whose process has a cyclical tendency due to regulations

Consider the group 8PSDr8. It is the sequences 6-7 that allow the return from the exploration phase to the planning-implementation phase. For group 8PVDr7, different round trips are materialized by steps 2-3; 4-5-6-7; 11-12. For these three groups, the regulations make it possible to move from an exploration phase to a planning-implementation phase rather than the reverse.

Processes not related to a normative model

All the other groups present processes whose tendency is not similar to the normative models mentioned above, that is to say that the course of the model is made according to a sequence of phases which exceeds the scope of those allowed by the two preceding models. For certain groups, these sequences are directly ensured by the students, i.e., without going through regulations.

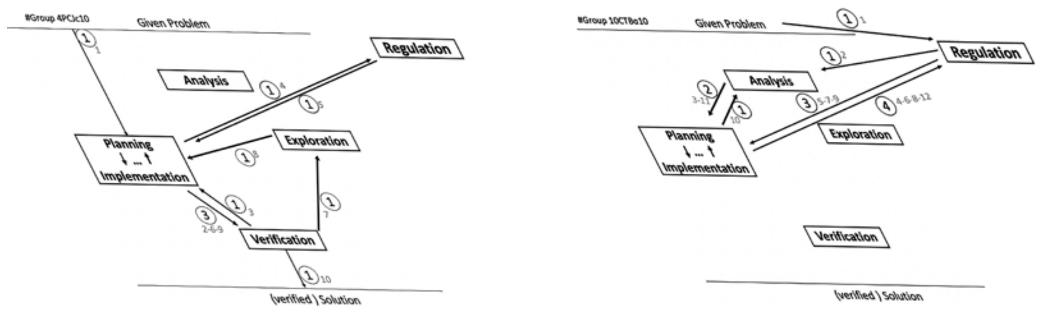

Figure 14. Groups whose process has a tendency not related to a normative model

This is the case, for example, for the sequences numbered 3 and 7 in group 4PCJc10, which highlight that students move from a verification phase to a planning-implementation phase (3) or from a verification phase to an exploration phase (7). For the 10CTBa10 group, it is step 10 that reveals the sequencing of a planning-implementation and analysis phase. For other groups, these sequences are made possible by the regulations. Thus, steps 9-10 of groups 4PCJc14 and 10CTBa7 (Figure 15) show that the students' work has moved from a verification phase to a planning-implementation phase via regulation.

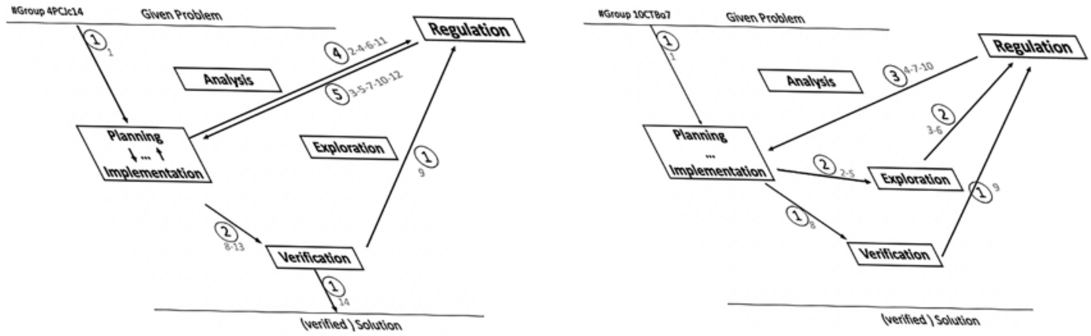

Figure 15. Groups whose process has a tendency not related to a normative model due to regulation

In conclusion, the enriched model I propose is operational to describe students' resolution processes in the case where the teacher intervenes. It allows us to identify the same trends as the model of Rott et al. (2021) when there is no teacher intervention. We thus counted a total of 3 groups whose processes have a linear tendency, 6 groups whose processes have a cyclical tendency in the strict sense of Schoenfeld, and 17 groups escaping these two tendencies. These 17 groups still represent two thirds of the corpus for which teacher interventions are recorded, which is very substantial. This reveals the limitations of normative models, particularly when we seek to describe the actual work of students in the classroom. However, this model has allowed us to highlight all the diversity and contingency that students' solving processes in class can present.

Moreover, I note that one type of process identified by Schoenfeld (1985) does not seem to appear in my analyses. This will be explained in the next section.

A Type Of Problem Solving Process Absent

Schoenfeld (1985) identified a type of problem-solving process that he called "wild goose chase" and described it as follows "The students [...] embarked on a series of computations without considering their utility and failed to curtail those explorations when (to the outside observer) it became clear they were on a wild goose chase." (p. 316) An operationalization of this type of process is proposed by Rott et al. (2021):

> A process is considered by us to be a "wild goose chase", if it consists of only *Exploration* or *Analysis & Exploration* episodes, whereas processes that are not of this type contain *planning* and/or *verifying* activities (only considering content-related episode types). (Ibid, p. 14)

None of the groups in my corpus can be classified a priori in this type. At first look, this result may seem surprising since my corpus, composed of 33 groups divided into six classes, represents a sample that seems sufficient to observe the diversity of processes. Also, it is unlikely that there are no such groups in my data but rather that the characterization proposed by Rott et al is not operational to identify them among my data. Indeed, the fact that my experiments take place in classrooms with interventions by a teacher who manages the session leads me to believe that this type of student manifests itself through other behaviors. We hypothesize that the teachers in our experimental set-up have identified these students or groups of students who might have tended to waste their time on calculations without considering their usefulness and to get lost in false leads and that they have intervened with these groups. Thus, it would be the teachers' interventions that impact the students' behavior and prevent me from identifying the processes that might characterize this "wild goose chase" profile. That said, if I cannot identify these groups of students, I can characterize another type of process. Indeed, five groups, 4PCJc14, 8PVDr8, 10CTBa10, 10CTBa12 and 10CTBa14, present processes that are quite remarkable from the point of view of the significant number of connections, almost always consecutive, between a planning-implementation phase and the regulation provided by the teacher. For example, for the 10CTBa14 group, the description allowed by our model shows the sequence of 6 back-and-forth movements between a planning-implementation phase and the regulation phase. These round trips are consecutive since they correspond to the sequences numbered from 4 to 15. We record 5 round-trips of the same type for the 10CTBa12 group (sequences numbered from 2 to 11) and 4 for the other three groups.

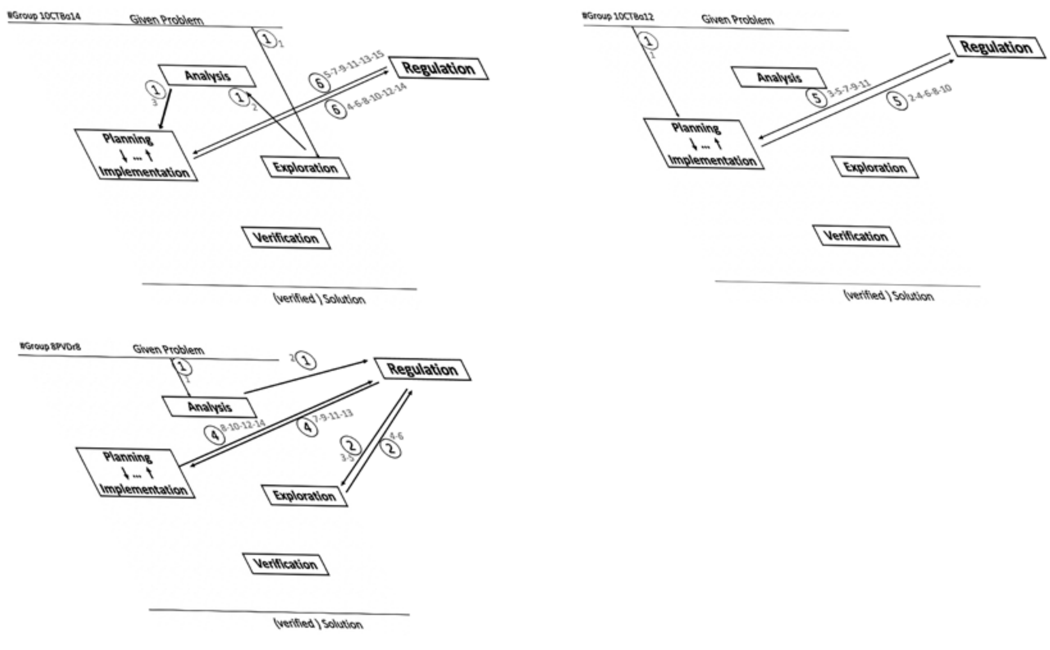

Figure 16. Examples of groups with high teacher involvement

Thus, the common point of these five groups is the important presence of the teacher through a large number of regulations. It seems that these groups need to be supported in their work, which would explain why their process is punctuated by the teacher's numerous interventions. However, the fact that regulations are linked to a planning-implementation phase indicates that after each regulation, the students are able to implement a track that has emerged from the regulation, even if it is not known, a priori, whether the track is proposed by the teacher, by the students or if it is co-constructed. This finding is all the more significant since these five groups belong to three different classes and therefore three different teachers.

Relationship Between Problem Success And Model Type

In this section, the distribution of the different groups according to the type of model and to the success of the problem is examined.

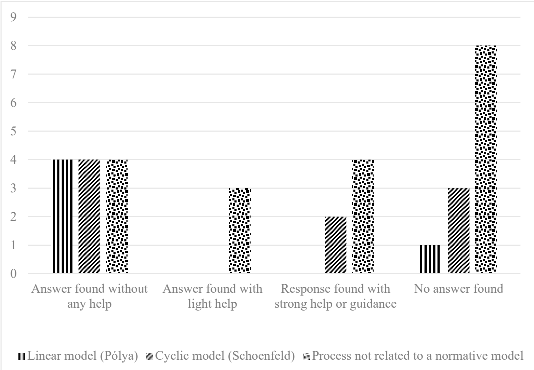

Table 3. Distribution of the groups according to the type of model to which it refers and according to the success of the problem.

This distribution indicates that four of the five groups whose process is similar to Pólya's model solved the problem and, what is more, without help. Moreover, these four groups were among those who found the answer in the shortest time (1-15 minutes) compared to the other groups who found it in 20-30 minutes. These indicators suggest that the problem may have seemed simpler to them. This confirms that Pólya's model is better suited to account for the work of groups for whom the problem did not offer much resistance.

Among the groups that did not manage to solve the problem, i.e., those for which the resistance was sufficiently strong, I note that processes not related to a normative model largely dominate (12 groups out of 18).

CONCLUSION

In my research, I aim to characterize the processes used by students when they solve mathematical problems in class. To do so, I first used Schoenfeld's video analysis framework to divide the students' work into episodes. An adaptation of this framework was necessary with the introduction of a regulation episode in order to take into account the moments during which the teacher intervenes with the students. This first analysis shows a very large disparity, particularly in terms of the number of episodes. In order to be able to interpret this disparity, I chose to reduce the complexity by focusing on the episodes relating to the students' work and by analysing their sequence. I thus operationalized the descriptive model of resolution processes developed by Rott et al (2021). This operationalization allowed us to identify three main trends with respect to the 7 groups that do not present a regulation episode: linear processes (in the sense of Pólya), cyclic processes (in the sense of Schoenfeld) and non-linear processes. This being said, a limitation of this model, inherent to my experimental data, is the complexity brought by the teacher's intervention in the students' work. To take this into account, we have enriched the model of Rott et al. by inserting a phase that is likely to be connected with all the other phases of the model. When I operationalized this enriched model, I was able to characterize the resolution processes according to the same three tendencies.

Finally, I was able to highlight groups of students whose resolution process is marked by numerous back and forth movements with a regulation phase. Most of the time, this back and forth is done with a planning-implementation phase. This leads me to wonder whether the research is left to the groups of students or whether they apply ideas proposed by the teacher.

I could realize that focusing on the external dimension of the problem-solving process is interesting but obviously not sufficient. This is why I complete this work by analyzing the internal dimension and more precisely the heuristics and knowledges that students invest in problem solving. The exploration phase is, according to Schoenfeld (1985), the heuristic core of problem solving. This invites me to investigate whether there are links between the different phases of the descriptive model and the heuristics used.

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