Guest editorial

EDITORIAL FOREWORD FOR THE SPECIAL ISSUE OF HIROSHIMA JOURNAL OF MATHEMATICS EDUCATION ON MATHEMATICAL PROBLEM SOLVING AND PROBLEM POSING

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The theme addressed in this special issue, *Mathematical Problem Solving and Problem Posing*, arose from the contributions that were presented and discussed during the sessions of the Topical Study Group (TSG) on *Mathematical Problem Solving and Problem Posing* at the 14th International Congress of Mathematics Education (ICME) held in Shanghai in 2021. The papers presented in this issue continue the discussion we had during the meetings of the TSG. This special issue affords some form of platform for more interested researchers to extend and refine the discussion and engagement on this important research and practicing area in mathematical problem-solving themes.

Although problem solving and problem posing are closely related, research on problem solving has gone through many decades of significant development while problem posing remains a relatively new field. This collection contains three papers on problem solving and four on problem posing. The problems on problem solving, although mainly built on the early works of Polya and Schoenfeld, have expanded beyond their seminal work on this field.

BOS and BOGAART (2022) advocate the use of heuristic trees, a form of digital tools as a scaffold to foster students' independence and to achieve problem-solving competencies. In order to scale up the use of heuristic trees, teachers must be able to design a heuristic tree, which in itself, is a challenging task. In analyzing the difficulties that teacher participants in a professional development course encountered in designing heuristic trees, Bos and Bogaart collated a list of mistakes, and provided the design principles of heuristic trees.

ABDULLAH and ABBAS (2022) conducted a study on teachers' exploration of using Graphic Organizers (GO) for teaching problem solving in the primary mathematics classroom. A group of teachers participating in the professional development workshop was introduced to the use of GO to help primary school students in solving word problems. They implemented the GO and assess their students' learning. The teachers' reflections on the affordances and challenges of the GO were also discussed.

Some of the models of the problem-solving processes are linear as proposed by Pólya's model and others are cyclical such as the Schoenfeld's model. However, the large majority of descriptions of the problem-solving processes do not fall neatly into these two models. FAVIER (2022)'s work on a characterization of the processes shown by the students in problem solving within the context of a classroom setting, provides one example of this alternative model for teaching problem solving. The analysis of the empirical data collected in this study led to the enrichment of the Rott et al. (2021)'s descriptive model of problem-solving processes , with an additional dimension involving the interactions between students and teacher.

Ramírez et al. (2022) proposed a heuristic strategy for enhancing problem posing in the mathematics classroom. The strategy is supported by a cognitive framework consisting of six stages (in short, SCASV+T framework). The six stages are Selecting, Classifying, Associating, Searching, Verbalizing, and Transforming. The authors presented the didactic considerations in implementing the heuristic strategy. In particular, the authors illustrated with a problem of elementary geometry how new problems can be posed based on an existing problem.

WANG and WANG (2022) carried out a textbook analysis of the mathematical problems and problem-solving tasks found in six series of mathematical textbooks in China. They focused on the distribution of the number, types of problems from the perspective of historical comparison, the types of problem-posing tasks and the distribution of these problem-posing tasks across various mathematical content strands. It was found that since the 1990s, a large number of mathematical problems has been included and evenly distributed throughout the textbooks. With the recent emphasis in problem posing, there had been an increase in the number of problem-posing tasks. But these tasks were not evenly distributed throughout different topics. Moreover, the total of problem-posing tasks was found to be still very low.

LUO et al. (2022) conducted an international comparative study between students from two regions in China and the United States, investigating the rural elementary students' thinking about division by analyzing the problems that they posed about division. Bruner's paradigmatic and narrative modes of thought was used as a framework for this study. The problems posed by the students about division were only about partitive and equal groups division, with none from either country posing problems on array or area. Most of the context of the problems posed were on food, though with cultural variants across the two countries.

CHUA and TOH (2022)'s exploratory study on secondary school students' problem posing, used two types of problem-posing tasks, one free-posing and one semi-structured task. The authors found a variety of problem-posing strategies used by the students, with the free-posing task elicited a wider variety of problem-posing strategies from the students. The number of problems-posed by the students was also not dependent on the students' achievement type. The authors further discussed the implications of the findings for teaching problem-posing in the mathematics classroom.

We hope that this collection of papers could contribute to the contemporary knowledge of problem solving and problem posing. The Editorial Team also like to raise the readers' attention that while this issue is being prepared, another set of authors from the same TSG of ICME14 have been invited to contribute to chapters of another book on problem posing and problem solving, which is expected to appear sometime in 2023.

From the Editorial Team

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