A PRACTICE REPORT ON MATHEMATICAL MODELLING EDUCATION FOR HUMANITIES AND SOCIAL SCIENCES STUDENTS

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Abstract

This is a practice report on our teaching practices of mathematical modelling education for humanities and social sciences students at a Japanese university. Our practices were initiated in order to respond to a request from social sciences and psychology departments, and to the growing social demand for such education in Japan. It has been a challenging task because many of these students are not good at mathematics, and some have math anxiety. In this report, we will reflect on our teaching practices over a nine-year period, including the preparation phase, and report our findings.

Key words: university mathematics education, mathematical modelling education, humanities and social sciences students, math anxiety

INTRODUCTION

Mathematics supports our lives in various areas. Mathematical models and statistics are becoming important tools in many research fields. From the beginning of this century, there has been a growing emphasis on fostering students' ability to use mathematics in problem-solving situations in the real world. The Programme for International Student Assessment (PISA) study by the Organisation for Economic Co-operation and Development (OECD) is the most famous and has affected primary and secondary school curricula worldwide. Although the term "mathematical literacy" is used in PISA's assessment framework (OECD, 2017), the emphasis is on the mathematical modelling process and its activities. At the university level, mathematics education for non-specialists has received considerable attention in recent years. As such, mathematics education for engineering students has reveived attention mainly in the last century. Recently, mathematics education for students in other majors has also gained attention. In European countries, mathematics in other disciplines, such as engineering, physics, biology, economics, etc., is being studied (Alpers et al., 2013; Feudel, 2018; Gonzalez-Martin & Hernandes-Gomes, 2019; Viirman & Nardi, 2019, etc.). In the United States and Japan, such education is often discussed in the context of general education; hence, mathematics education is discussed not only for students in science, technology, engineering, and mathematics (STEM) majors but also for students in social sciences and humanities majors. In the United States, the term "quantitative literacy" (QL), which is also called "numeracy" or "quantitative reasoning" (QR), is used to refer to such education. It is defined as "a "habit of mind," competency, and comfort in

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working with numerical data" (Association of American Colleges and Universities [AAC&U], 2009) and the AAC&U (2009) provides the Quantitative Literacy VALUE Rubric for the assessment of QL. There are reports on QL and QR education (e.g., Rocconi et al., 2013), some of which deal with math anxiety among students in social sciences and humanities majors (e.g., Henrich & Lee, 2011). In Japan, it has often and long been claimed that all of university students, including humanities and social sciences students, should have mathematical abilities at a certain level. In the discussion on mathematics education for non-specialists, the term "mathematical literacy" has been used in Japan since the 1980s (Nagasaki & Abe, 2007). Subsequent to the OECD's PISA, it is used in the sense embodied in that framework, but it includes university mathematics education in the Japanese context. Very recently, the Japanese government launched a program to promote tertiary-level education in artificial intelligence, mathematical sciences, and data sciences, with the aim of increasing the number of experts in those fields and promoting a beginner-level education in those areas for all university students. Although university education systems differ among countries, they share the same emphasis on mathematical modelling activities in mathematics education for non-specialists.

In the contexts of the United States and Japan, mathematics education for students majoring in social sciences and humanities fields has been recognized as a challenging task. This is because many of these students are not good at mathematics, and some have math anxiety, making it imperative in mathematics education to reduce their dislike and fear. Fujii (1994) reported that social sciences and humanities students experienced more math anxiety than STEM major students did. There have been various approaches to address the issue of teaching mathematics to math anxious students. For example, in a study on the subject, Henrich and Lee (2011) incorporated a service-learning component into a QR course primarily taken by humanities majors. During our teaching practices from 2011 to 2019, we attempted to address this issue by developing design principles focused on mathematics education for humanities and social sciences students. In our previous studies (Kawazoe et al., 2013; Kawazoe & Okamoto, 2017; Kawazoe, 2019), we demonstrated that a course based on our design principles could increase students' interest in mathematics and their self-confidence in their mathematical thinking skills, and could change their view of mathematics. However, we have not investigated the course design from a long-term perspective. In this paper, we would like to extract knowledge about the design of mathematics education for humanities and social sciences students by reviewing our teaching practices over the past nine years. More precisely, we address the following research questions: (1) Is each item comprising our design principles sufficiently robust for long-term use?; (2) Regarding any modifications to the design principles, what changes were made, and why?

DATA AND METHODOLOGY

The mathematics course investigated by the present study consists of two successive one-semester mathematics classes, a spring semester class and a fall semester class, for humanities and social sciences students in their first year at a Japanese university. Both are two-credit classes that meet for 90 minutes each week for 14 weeks, followed by an examination period. The course began in the 2012 academic year, after a one-year pilot course in the 2011 academic year. We stored various data concerning the course during the period 2011 to 2019, including syllabi, lesson plans, paper-based worksheets used in each lesson, homework (paper-based, online), students' self-reflections after each lesson and at the end of each semester, questionnaires administered at the beginning and end of each semester, and the log data of e-learning materials. We conducted a retrospective analysis with these data, chronologically describing our teaching practices over the past nine years, including the preparation phase.

REFLECTION ON TEACHING PRACTICES INCLUDING THE PREPARATION PHASE

In 2010, our university decided to offer mandatory mathematics and statistics courses to humanities and social sciences students beginning in the 2012 academic year. Course development started in 2010. The author contributed to the development of the mathematics course. In the following subsections, we reflect on the mathematics course teaching practices, including the preparation phase.

Course preparation: Oct. 2010–Mar. 2012

In the preparation phase, the author, as a mathematician, partnered with a cognitive psychologist to develop the course. We developed a mathematics course consisting of a spring semester class and a fall semester class. Each class was designed as a two-credit class that would meet for 90 minutes each week for 14 weeks, followed by an examination period.

Determining the course objective: The course objective was determined according to the following four factors: social demand, the demand from the departments to which the mathematics course is offered, mathematics teachers' opinions, and students' mindset (i.e., their attitudes toward mathematics and mathematical knowledge). To meet social demand, mathematics education should be offered to foster students' ability to use mathematics in real-world situations. Regarding demand from within the departments, students need to acquire sufficient mathematical knowledge for learning about statistics and the mathematical models used in their disciplines. Considering these requirements, the course objective was to develop basic mathematical knowledge and appropriate attitudes for the application of mathematics in the social sciences as well as in real-world situations.

Designing the course: There were two main ideas regarding course design. The first was the order of instruction. In a traditional mathematics class, definitions, formulas, exercises, and applications appear in this order. We changed this order and designed each lesson to begin with an applicational problem in a real-world situation. Mathematics is invisible at the beginning; it only appears after mathematizing the problem. Mathematical exercises come last. Initially, a real-world problem is presented, with the aims of attracting students to the lesson, communicating the importance of mathematics in the real world, and familiarizing students with the mathematical modelling process (Figure 1). The second idea concerned the teacher's language during instruction. Many students in the humanities and social sciences do not have a sufficient understanding of teachers' mathematical explanations in class. Our idea for improving this situation was to explain mathematical concepts and procedures in both mathematical language and everyday language. During the pilot course in the 2011 academic year, we examined the effectiveness of teaching materials and instruction methods and improved them based on our findings.

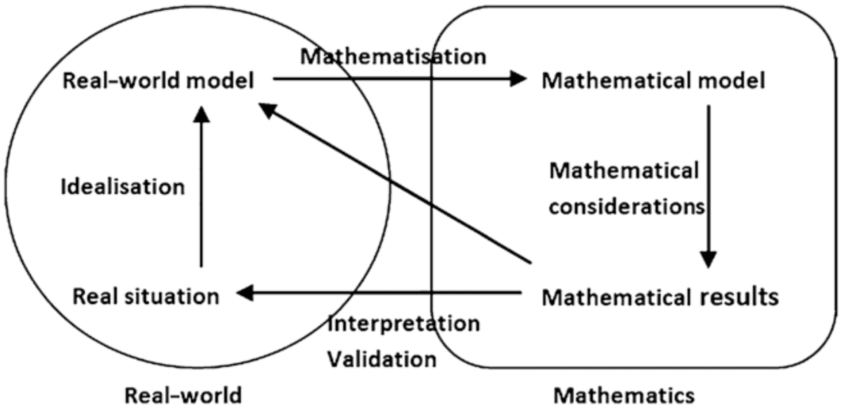


Figure 1: Modelling process from Kaiser-Meßmer (1986) and Blum (1996)

Design principles developed during the preparation phase

The following design principles were developed based on the result of the pilot course (Kawazoe et al., 2013; Kawazoe & Okamoto, 2017):

(1) *Design lessons based on the mathematical modelling framework*: In a lesson designed based on this principle, a real-world problem is presented at the beginning. Then, students engage in mathematization activities and solve the given problem by using mathematics. Following this principle, the course is implemented as a mathematical modelling education.

(2) *Choose topics and contexts with consideration of the mathematical knowledge students encounter in their real life and the situations in which they encounter it*: According to this principle, problems presented during classroom activities should be connected to everyday contexts, so that students can understand when, how, and to what the mathematical knowledge is applicable in real-world situations.

(3) *Connect mathematical language to everyday language*: It is necessary to explain topics in both mathematical and everyday language in order to help students who have difficulty understanding mathematical language comprehend mathematical concepts and methods as well as their value.

(4) *Engage students in group rather than individual activities*: Most students view mathematics learning as individual learning, and they fear it because they are not confident about solving mathematical problems by themselves. This principle aims to reduce students' math anxiety and increase their motivation to learn mathematics.

(5) *Present problems in multiple contexts associated with the same mathematical knowledge*: Mathematics instruction using a real-world situation is sometimes criticized because students' learning is limited to the situation or context used in class. This is the so-called *the situatedness of learning* (cf. Sfard, 2014), which this principle aims to overcome.

(6) *Connect different mathematical knowledge by using different mathematizations of the same problem or by using mutually related contexts*: This principle aims to promote the structurization of mathematical knowledge in students' minds.

(7) *Design paper-based worksheets that follow students' comprehension process and use them as tools for formative assessment*: A course developed with the above principles would be more effective if students'

activities were driven by well-designed worksheets. Worksheets are used to assist students' understanding and also help students assess their learning process by themselves.

As stated in our previous studies (Kawazoe et al., 2013; Kawazoe & Okamoto, 2017), the above principles originated from the four perspectives of learning environments developed in learning science (Bransford, 2000, Chapter 6): student-centered, knowledge-centered, assessment-centered, and community-centered (Figure 2). Principles (1), (2), (5), and (6) originated from discussions about mathematical literacy (cf. Sfard, 2014) and mathematical modelling (cf. Kaiser, 2014). Principles (5) and (6) are aimed at *decontextualizing* and *structuralizing* students' knowledge, respectively, because teaching mathematics in real-world contexts has been found to restrict students' knowledge to the current learning context (cf. Sfard, 2014). Although the above principles do not explicitly state this, the active use of information and communications technology (ICT) tools to perform calculation is also important in our course design.

Figure 2: Four perspectives of learning environments (Bransford et al., 2000, p.134, Figure 6)

Course topics

The topics taught in the spring semester class are as follows: systems of linear equations/inequalities (linear programming), number sequences (savings, loan payment, pharmacokinetics, model of addiction), matrices and vectors (spreadsheets, social network analysis), functions (mental rotation, pharmacokinetics, bacterial growth, pandemic, periodic movement of electric demand), and probability (lottery, disease examination, birthday paradox, Bayesian estimation). The topics taught in the fall semester class are as follows: eigenvalues/vectors (population dynamics, optimal distribution), derivatives (innovation diffusion, population growth, logistic function, marginal profit, optimization), integrals (speed and distance, accumulated radiation level, standard normal distribution), and multivariable functions (loan simulator, formulas for estimating vital capacity).

We provide some examples of the problems presented in the course. The following examples were developed during the pilot course.

Problem 1. You have begun to take hay fever medication. You have to take one tablet every 12 hours. If you continue to take this medication over a long period, what amount of fexofenadine hydrochloride, the medication's main component, will remain in your body? Consider the amount of fexofenadine hydrochloride immediately after taking the drug every 12 hours. Each tablet contains 60 mg fexofenadine hydrochloride, and approximately 40% of fexofenadine hydrochloride remains in the body after 12 hours.

Problem 2. Due to the spread of foot-and-mouth disease in Miyazaki Prefecture in Japan in the spring of 2010, a large number of livestock were killed. The following table summarizes the changes in the cumulative number of livestock suspected to have the disease over 18 consecutive days, starting from April 20, 2010. Analyze the spreading of the disease by using a semi-log plot, and create a mathematical model of it by using an exponential function.

Day	1	2	3	4	5	6	7	8	9
The number of suspected livestock	16	202	266	386	386	1111	1111	1111	2893
Day	10	11	12	13	14	15	16	17	18
The number of suspected livestock	2943	4416	8298	9056	9096	28720	35247	47556	64827

Problem 3. You have tested "positive" for a certain disease. One in 1,000 people is said to have this disease. The test is said to be 99% accurate; that is, 99% of people who have the disease test positive, and 99% of people who do not have it test negative. Estimate your risk of having the disease.

Problem 4. O University has decided to start a bicycle rental service because of its huge campus. Two parking areas are located beside the main gate Nakamozu-mon (Parking Area A) and in front of lecture building B3 (Parking Area B). Students can drop bicycles off in either of the two parking areas. A monitoring investigation showed that bicycles in both parking areas move as shown in the following table every week. The numbers in the table are displayed as percentages. How should bicycles be divided between the two parking areas when the service starts?

	To Parking Area A	To Parking Area B
From Parking Area A	70	30
From Parking Area B	20	80

Problem 1 is presented in a lesson on the application of number sequences. Problem 2 is presented in a lesson on the application of functions (focusing on exponential growth). Problem 3 is presented in a lesson on probabilities. Problem 4 is presented in a lesson on eigenvectors and eigenvalues. It should be noted that Problem 4 is presented to students before they learn eigenvectors and eigenvalues. The lesson using Problem 4 is designed so that students can learn eigenvalues and eigenvectors together with their applications in the classroom activity in order to solve the problem. Similar to Problem 4, some problems are used to teach new mathematical concepts.

Teaching practices for the regular course: Apr. 2012–Feb. 2020

The course started in the 2012 academic year with four classes in each semester. Four mathematics teachers, including the author, taught the classes. The spring semester class is mandatory for all humanities and social sciences students, and approximately 300 students take the class every year. Until the 2017 academic year, the fall semester class was mandatory for students in economic majors. Beginning in the 2018 academic year, only the spring semester class was mandatory for all students. We have been trying to grasp the characteristics of the classes using a questionnaire survey administered at the beginning of the spring semester. For example, the 2018 results showed that about 30% of students who took the course had been in the science-oriented course in high school, while the others had been in the humanities-oriented course. More than half indicated that they were not good at mathematics, and about 40% indicated that they did not like mathematics. The percentages may change from year to year, but this is the average trend.

An Issue encountered at the start of the course and our improvement strategy: When the course began in 2012, we had to consider methods for sharing lesson ideas and tips among four teachers. We developed a textbook (Kawazoe & Okamoto, 2012) and paper-based worksheets for use in all classes, but these did not seem to be sufficient. We decided to develop a document of "lesson plan" for each lesson, which is popularly used in primary and secondary schools in Japan, and to use it as a tool for sharing ideas and tips of a lesson. Using lesson plans, we could control the quality of the classes.

During the eight-year practices in this period, most of the course topics did not change, except for that the Lagrange multiplier method and some other topics were added in the 2015 academic year. On the other hand, students' activities, homework, and assessment tools have changed.

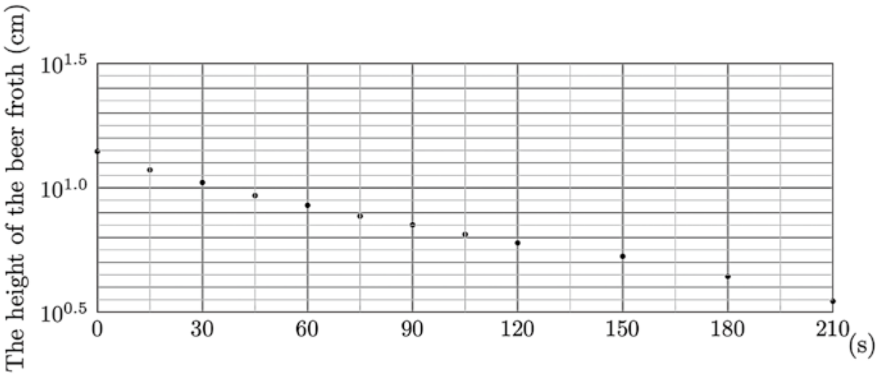
Adding new topics: In 2014, the faculty of economics requested that the course incorporate additional topics. The faculty wanted their students to be exposed to determinants, logic, and the Lagrange multiplier method in their first year. Regarding determinants and the Lagrange multiplier method, they said that it was enough to teach determinants of 2×2 matrices and the Lagrange multiplier method applied to an easy example of functions with two variables. We added exercises on determinants and logic to a worksheet on systems of linear equations and a worksheet on probabilities as optional exercises, respectively, beginning in the spring semester of 2015. Regarding the Lagrange multiplier method, we developed a new 90-minute lesson using the following problem.

Problem 5. Many buildings have water tanks on their roofs, some of which are cylindrical tanks. When looking at a cylindrical tank from the side, it often appears to be a square. In order to consider the reason, solve the following question: "You want to design a 2,000π m^3 cylindrical tank, minimizing the surface area to save materials. Find the height and the radius of the tank."

Problem 5 was created based on an example from a textbook written by Kawanishi (2010). We believe that this easy problem is suitable for introducing the Lagrange multiplier method, although the problem can be solved without use of the method. The lesson was implemented as the final lesson in the fall semester, replacing a 90-min lesson devoted to learning formulas for integration by substitution and by parts. With this change, lessons on integration have put more emphasis on the meaning of integration rather than on calculation. This change has been applied since the 2015 academic year.

Continuous development of new problems: As mentioned in the design principles, we believe that it is important to present problems in multiple contexts. We developed many problems in multiple contexts during the preparation phase, but for some topics, the variety of contexts was restricted. Moreover, problems taken from real-world situations are often associated with a specific time period, and in particular, problems that entail predicting the future lose their connection with reality after the passage of the year indicated in the problem. In addition, we must create new problems with different contexts for the final examination in each semester because students can access the problems given in past examinations. Hence, we have attempted to continuously develop new problems. Here, we present an example. Similar to Problem 2, Problem 6 is on a semi-log graph, but it is in a different context. Problem 6 was used in the final examination in the spring semester of 2019.

Problem 6. It is known that the volume of beer froth decays exponentially with time. Physicist Leike (2002) confirmed this fact by measuring changes in the height of beer froth in a cylindrical glass over time and was awarded the 2002 Ig Nobel Prize[1] in physics. The graph shown below is a semi-log graph representing the results of an experiment using Augustinerbräu München. The horizontal axis represents time (s), and the vertical axis represents the height (cm) of the beer froth in the glass. Answer the following questions: (1) Explain why the height of the beer froth can be considered to decrease exponentially, based on the graph, and (2) create a mathematical model representing the height (cm) of the beer froth at the time t (s) by using an exponential function.



Creating new problems for final examinations is a very hard task for teachers, and it gets harder year by year. Hence, it can be said that creating new problems for final examinations is related to the sustainability of the course.

Changing the balance between group and individual activities: In the early years, we implemented individual and group activities during each lesson. Each teacher has their own method for creating groups. Usually, each group consists of three to four students, but some teachers prefer students to work in pairs. Although we encouraged students to interact even during individual activities, it did not go well because students' mathematical abilities differed. Very capable students could finish the task in a short time, while less capable students could not understand how to do and could not finish the task in time. We then gradually reduced individual activities and increased group activities in each lesson. We also tried to change the way groups were organized so that capable students were distributed across all the groups. In order to accomplish this, groups are organized based on the results of a questionnaire administered at the beginning of the course in which students are asked about their attitudes toward mathematics and their high school learning history. Then, group activities became to be aimed at helping all students understand the meaning of mathematical procedures. Capable students are expected to teach others in a group setting. Individual activities are assigned as homework. We verified the effectiveness of group activities by analyzing the results of a self-report questionnaire administered at the end of the spring semester of 2019. The questionnaire asked the students to write up to three of the most important things that they learned during the semester. In response to this question, 15 of the 70 students in the author's class mentioned group activities or learning with others as one of the three important things that they learned. We also asked them to freely write their thoughts and

[1] The Ig Nobel Prize is a parody of the Nobel Prize. It is awarded annually since 1991. On its official website, the award is said to honor "achievements that make people LAUGH, then THINK."

comments about the class. Sixteen students described group activities as effective in their comments. The following are examples of students' comments pertaining to group activities:

Student 1: By exchanging opinions during group work and so on in order to derive formulas, I could gain a deep understanding of mathematics.

Student 2: By solving in a group, I got explanations from others about things that I could not understand, I felt that there were few cases where I could not understand something during the class, compared to when I was in high school.

Student 3: In mathematics, I found that I get more creative thought and have more fun when we solve in a group than when we do it alone.

(These comments were written in Japanese and translated into English by the author.)

The above comments indicate that increasing the frequency of group activities contributed to reducing the number of students who could not understand the class.

Implementing e-learning homework: In the early years, homework was paper-based, consisting of modelling tasks and basic computational tasks. In the 2018 academic year, we developed e-learning materials consisting of basic computational tasks in order to enrich students' after-class learning and increase its effectiveness. We have been using these materials since then. E-learning materials have been implemented on Moodle, and some have been implemented as STACK question type (The STACK project, 2020). Students can access the materials at any time and receive the immediate feedback. Students can try them multiple times before the deadline, and the highest score is adopted. Regarding e-learning usage, it has been observed that students made repeated attempts until they succeeded, and the percentage of reviews in students' after-class learning has increased. Examples of e-learning materials are shown in Figure 3 and 4.

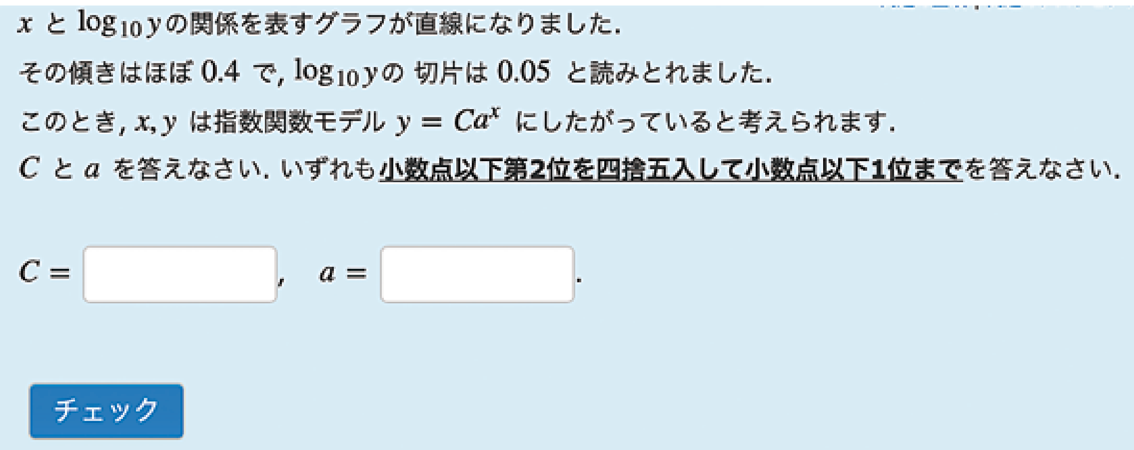


Figure 3: An example of e-learning material: "Assume that a graph representing the relation between x and $\log_{10} y$ is a straight line. Its slope is about 0.4 and a $\log_{10} y$-intercept is read as 0.05. Then, the relation between x and y can be modeled with an exponential function $y = Ca^x$. Find C and a."

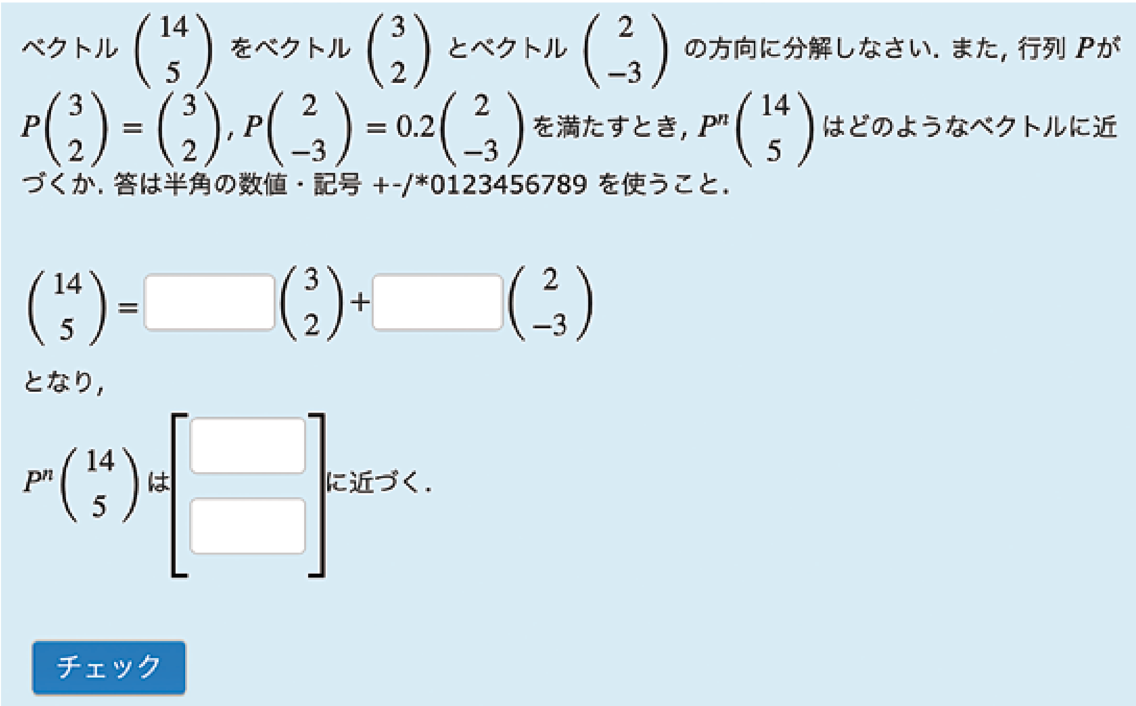


Figure 4: An example of e-learning material: "Decompose a vector $\begin{pmatrix}14\\5\end{pmatrix}$ into two directions given by $\begin{pmatrix}3\\2\end{pmatrix}$ and $\begin{pmatrix}2\\-3\end{pmatrix}$. When a matrix P satisfies $P\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}3\\2\end{pmatrix}$, $P\begin{pmatrix}2\\-3\end{pmatrix}=0.2\begin{pmatrix}2\\-3\end{pmatrix}$, what vector does $P^n\begin{pmatrix}14\\5\end{pmatrix}$ get close to when n becomes larger?"

Evolution of assessment tools: Students' mathematical knowledge can be assessed using a paper-based test. To evaluate teaching practices, it is important to assess changes in students' attitudes and their mathematical habit of mind. We developed a rubric that we have been using as a self-evaluating tool for students since the 2015 academic year. Development of an effective assessment tool is still in progress (e.g., Kawazoe, 2019) and remains a challenging task.

Another sustainability issue: In 2018, we faced a serious problem due to the retirement of one of the four teachers. We could not find another suitable teacher within our university, so three teachers had to teach the four classes in each semester until a new teacher was hired in April in 2019. This made us aware that teacher development is another important issue for the sustainability of the course.

FINDINGS FROM TEACHING PRACTICES AND CHALLENGES FOR THE FUTURE

As a result of the nine-year practices described in the previous section, we obtained the following findings. First, our design principles and topics were proved to be viable in the long run. As stated in the previous section, the basic framework of the course and the content of each lesson have remained almost the same since 2012, except for the change due to requests from the faculty of economics. Although our lesson design was very different from mathematics classrooms in Japanese high school, students' self-reflections collected via worksheets showed that many were satisfied with exposure to real-world problems to which mathematics can be applied. Many students who were not good at mathematics described a reduction in their negative feelings about mathematics in their self-reflection comments at the end of the semester. Second, it was found that a lesson plan is a useful tool for sharing ideas and tips of lessons among teachers. The main component of a lesson plan is a timetable containing instructions for students' activities, contents on black board, and teacher's questions. Lesson plans have helped three teachers, other than the designer of the course, conduct their own classes smoothly. Additionally, when a new teacher joined as a lecturer of the course in 2019, lesson plans helped him start his first year of teaching. This confirms that lesson plans help a course designer to make other teachers understand ideas regarding lesson aims and strategies for asking questions in a classroom. Third, group activities were found to be more effective than we previously thought. This is evidenced by the growing number of comments from students in their self-reflections about the advantages of group activities since the year when we started to increase group activities. Until the 2019 academic year, groups were fixed during the semester. In the fall semester of 2020, the author reorganized the groups on a trial basis, but due to the spread of the novel coronavirus (COVID-19), the class shifted to an online class, preventing verification of the effect of reorganizing groups on students' learning, which remains a future task. Fourth, the use of e-learning was found to enhance students' after-class learning. When computational tasks are given as paper-based homework, the number of problems is restricted owing to the space constraints of the paper and the time constraints of the teachers who grade them. Additionally, in the case of paper-based homework, students cannot receive immediate feedback because homework is submitted in the next week's class and is not graded and returned until a week later. With e-learning, it is possible to randomly assign problems from a vast problem pool or to randomly generate equations in a problem. Furthermore, students can repeat exercises as many times as needed and receive immediate feedback. From the teachers' perspective, using e-learning materials for computational tasks allows them to focus on grading tasks that assess students' thinking process, such as modelling tasks. These four findings are positive.

However, two issues need to be addressed. One is the need to develop an effective tool for assessing changes in students' attitudes and mathematical habits of mind. The author conducted a pilot study (Kawazoe, 2019) to assess these by analyzing students' self-reflection comments collected via worksheets. The results of the study indicated that students' awareness of the essence of mathematical thinking and knowledge could be found in their comments. However, extraction requires careful perusal of students' weekly comments, which is a hard work for teachers. Further studies are needed to determine whether this method can be established as an evaluation method and whether the evaluative efficiency can be improved. The other issue is sustainability, which contains two sub-issues. One is the need to continue creating new problems, especially for final examinations. This is getting harder each year, and we have not yet been able to find a solution. The other is the need to develop mathematics teachers who can teach the course. It is very difficult to find mathematics teachers who can teach the course presented in this paper. The main reason for this is that almost all mathematics teachers in Japanese universities are pure mathematicians. Establishing strategies for developing mathematics teachers who can teach such courses is an emergent task pertinent to the sustainability of the course.

Finally, we note on our teaching practices in the 2020 academic year amidst the COVID-19 pandemic. Our university requested teachers to give their lectures in the spring semester on an on-demand online basis. The course presented in this paper was heavily affected by it, because group activities cannot be implemented in an on-demand video lecture format. We were apprehensive of students' reactions to online course delivery. Their comments revealed a strong desire to attend classes in person on campus, but on the other hand, students were very satisfied with the online course. Based on students' comments on each lesson, it was observed that they watched the videos repeatedly until they felt that they understood the content and that they engaged themselves in exercises at their own pace. These observations suggest that if we can give students enough time to perform exercises, they will be able to learn without math anxiety, even in an environment where only individual activities are available. This finding may provide an important insight into developing strategies for implementing group activities and individual activities in the classroom.

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