A SMALL-SCALE IMPLEMENTATION OF INQUIRY-BASED TEACHING IN A SINGLE-VARIABLE CALCULUS COURSE FOR FIRST-YEAR ENGINEERING STUDENTS

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Abstract

We report from the first iteration of a small-scale project introducing elements of inquiry-oriented education in a first-year engineering Calculus course. In four of the exercise sessions we introduced problem solving in groups, using problems designed to provide alternative viewpoints on central topics of the course, for example limits, differentiation and integration, and containing elements of modelling and numerical methods. The theoretical perspective underlying the design was commognitive theory. We discuss some of the problems used in the intervention, focusing particularly on the numerical differentiation and integration problems. We also report some observations made during the first two iterations of the project, and how these have fed into the continued evolution of the project.

Key words: Inquiry oriented teaching, calculus, engineering education, commognition, numerical differentiation, numerical integration

BACKGROUND

It is fast becoming an established fact that so-called "active learning" is beneficial for students when compared to traditional lecturing (Freeman et al., 2014). At the University of Gävle, where both authors worked at the time of the intervention described in this paper, for a few years now the first-year mathematics courses for the engineering programs have been run using a format called PLUSH (Preparation, Lecture, Unsupervised study, Seminar, Homework). The PLUSH format was designed to encourage students to engage more actively, without involving radical changes in the teaching, changes that might risk losing those aspects that have been highlighted as the strengths of the lecture format, such as motivating students and modelling mathematical practices (Pritchard, 2010). In the PLUSH format, before each lecture students are expected to watch short videos covering the essential aspects of the topic. This removes some of the pressure to "cover the content", making room for the lecturer to include more student-centered and conceptually oriented activities, such as clicker quizzes and small group discussion. After the lecture, students have two hours of unsupervised study in which to prepare for the afternoon seminar. However, these seminars were

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not working as well as we would have hoped. Rather than developing into discussion sessions revolving around the recommended exercises, due to student expectations and demands, these seminars typically turned into either continuations of the unsupervised study, but now with a teacher at hand for answering questions; or traditional exercise sessions, where the teacher does exercises at the board while students take notes.

In light of this, in the spring semester of 2019, for the Single Variable Calculus course we decided to implement aspects of an inquiry oriented (IO) teaching approach. There are successful undergraduate mathematics courses run entirely using an IO approach (e.g. Rasmussen & Kwon, 2007). However, not wanting to wholly remodel the course, we were instead interested in seeing if we could gain some of the benefits of the IO approach working on a smaller scale. We did this through devoting some of the seminar time to problem solving in small groups of 4-5 students each. Instead of working on selected exercises from the textbook, the students were asked to work on one or two larger problems for a longer period of time, about 45 minutes. These problems were designed to support and deepen their understanding of the central topics of the course, while also allowing them to practice working collaboratively, and presenting and arguing their mathematical ideas. In this paper, we present the ideas and theoretical considerations underlying the choice and design of problems, giving particular attention to one specific problem concerning numerical integration. Furthermore, we discuss some empirical observations, and how these have fed into the future iterations of the intervention. First, however, we give a brief overview of the context of the intervention.

CONTEXT – THE COURSE, THE STUDENTS, THE TEACHERS

The intervention took place in the context of a 7.5 ECTS credit course in Single Variable Calculus, aimed at first-year engineering students and covering the usual introductory calculus material: elementary functions, limits and continuity, derivatives, integrals, Taylor series, etc. The course has no prerequisites apart from the usual requirements on upper secondary mathematics, but due to its positioning in the study program, most students enrolling in the course will have taken courses in introductory and linear algebra. In addition to approximately 120 engineering students, a small number of prospective upper secondary teachers normally take the course every year. The course is taught over a nine-week period, with two days of teaching per week. These days are structured according to the PLUSH format, with a whole-class lecture in the morning followed by unsupervised study and four parallel seminars. At the time of the intervention reported here, four teachers were involved in the course: two research mathematicians (one of which is the second author of this paper) responsible for lectures and one seminar group each; a mathematics education researcher (the first author) teaching one seminar group, and a teaching assistant running one seminar group. Students self-selected to the different seminar groups, and were encouraged to try out different groups.

LEARNING AS ROUTINIZATION

Theoretically, the intervention was grounded in the commognitive theory of learning (Sfard, 2008), and particularly the notion of *routinization* (Lavie, Steiner & Sfard, 2019). From a commognitive perspective, mathematics is conceptualized as a discourse, and learning as the increased ability to participate in this discourse. The discourse of mathematics (indeed, any discourse) can be distinguished through four characteristics (Sfard, 2008, p. 133-135):

- *word use* – words specific to the discourse or common words used in discourse-specific ways, for instance Riemann integral, function, etc.;
- *visual mediators* – visual objects operated upon as a part of the discursive process, for instance diagrams and special symbols;
- *narratives* – sequences of utterances speaking of the description of objects, relations between and/or processes upon objects, subject to endorsement or rejection within the discourse, for instance theorems, definitions and equations;
- *routines* – repetitive patterns characteristic of the discourse, for instance methods of proof, of performing calculations and so on.

Recently, the notion of routine has been further developed and operationalized. "A routine performed in a given task situation by a given person is the task, as seen by the performer, together with the procedure she executed to perform the task" (Lavie et al., 2019, p. 161). Here, a *task situation* denotes "any setting in which a person considers herself bound to act" (p. 159), and a *task* "as understood by a person in a given task situation, is the set of all the characteristics of the precedent events that she considers as requiring replication" (p. 161). Thus, in problem design and analysis, we have paid particular attention to the kinds of routines students need to engage in when working on the problems, but also how these tie in to students' previously established routines.

PLANNING THE INTERVENTION

In addition to the underlying theoretical perspective, when designing problems for the sessions we were guided by several overarching principles. The sessions should complement the lectures and other teaching activities, taking different perspectives on central topics. Given that the course caters to engineering students, we wanted problems that included elements of modelling and numerical methods, which are not part of the curriculum in any of the obligatory mathematics courses in the three-year engineering programs at the University of Gävle, but which are very useful for future engineers (Alpers et al., 2013). At the same time, we wanted the problems to be flexible and doable with limited resources – we did not want to have to rely on access to computer labs or specialized software, for instance. Importantly, the problems should be grounded in research on the teaching and learning of calculus. One basic aspect of this was designing tasks that require creative rather than imitative or algorithmic reasoning, something that has been shown not to be the case for most standard calculus textbook tasks (Lithner, 2004). Moreover, we wanted problems that connected different parts of the course by making use of settings that could be revisited in later sessions focusing on other topics.

Given that one of our aims was to have the problem-solving sessions contribute to students' understanding of the central concepts of the course, we decided to focus the IO sessions on limits/continuity, differentiation, graph sketching, and integration. The sessions on differentiation and integration were designed to contain elements of modelling and numerical methods. The group work was intended to facilitate students' adaptation to the demands of the new form of teaching, and allow them to develop their collaborative and argumentative skills. In order to trace the progression in learning, we recommended the students to attend all IO sessions to benefit the most from the intervention. For the first trial, we decided to run the intervention in just two of the four seminar groups, to enable students to choose not to participate.

EXAMPLE PROBLEMS

The main problem used in the first IO session concerned the behavior of a function with an irremovable singularity at x=0, and involved estimating the limit of the function along different sequences converging to 0. Since we expected students to have limited experience of group problem solving, we designed the problem to provide a relatively high degree of scaffolding, dividing it into a series of sub-problems. In later sessions, to allow students greater independence, we gradually decreased the scaffolding. The main problem in the session on differentiation, following a brief warm-up problem introducing numerical differentiation and the notions of forward and central differences, concerned the travails of the Svensson family travelling to the airport and forgetting their passports. The problem involved numerical differentiation and the construction of a velocity-time graph from a given position-time graph of a function not given through a formula. This problem was later returned to in the session on integration, where students were asked to recover positions at certain times from the velocity-time graph. The session on graph sketching included a "Chinese whispers" problem on graph analysis, where a student verbally described a graph to a fellow student, who in turn drew a graph according to the description given. This new graph was then given to a third student, and the process was repeated. After a few iterations, the resulting graph was compared to the original, and similarities and differences were discussed. Apart from the follow-up to the differentiation problem, the main problem for the integration session involved numerical estimation of the volume of a vase. In what follows, we will present the numerical differentiation and integration problems in more detail, followed by some empirical observations.

The "Svensson's vacation" problem

The problem was formulated as follows:

> The Svenssons were going on vacation to Thailand. They packed their bags, got into their car and drove off for the airport. After a while, they started worrying that they might have forgotten their passports. They drove off at the next exit, and started rummaging through their bags. Sure enough, the passports were nowhere to be found. They turned around and drove back home. By now, they were in a bit of a hurry. When you're under stress, finding what you're looking for takes longer, but it turned out alright in the end, and the Svenssons got to the airport on time. Attached you see a graph (Figure 1) showing the time in minutes and the position of the Svenssons' car at the corresponding time, measured in kilometers from their house. Please answer the following questions:
>
> - What was the velocity of the car at times t=3; 10; 22; 55 minutes?

- What was the average velocity of the car in the time intervals [10,15] and [20,25] minutes?
- Sketch a graph showing the velocity of the car as a function of time. Does the graph look reasonable? Does it agree with the story of the Svenssons?

As briefly mentioned above, the session on definite integrals included a follow-up to the "Svenssons' vacation" problem. The students were given a graph of the approximation for the instantaneous velocity (Figure 2) and were asked to estimate the distance from home for Svenssons after 15 and 25 minutes respectively.

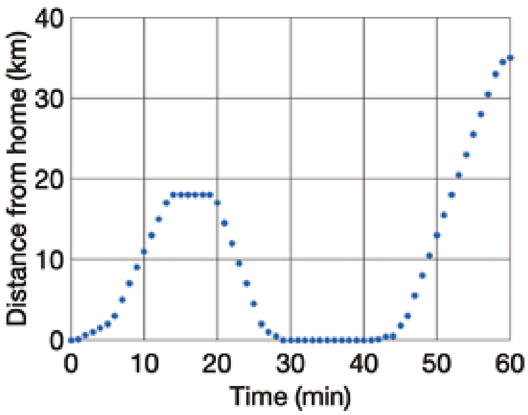


Figure 1: Position of the car at time t

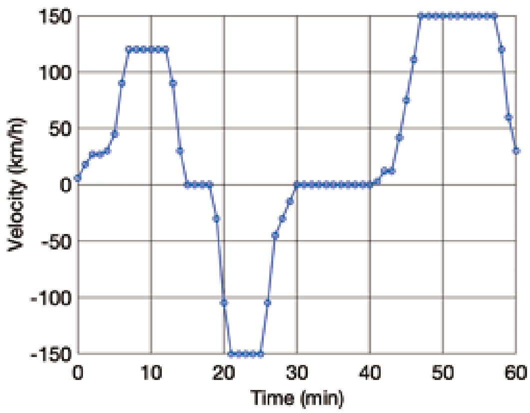


Figure 2: Velocity of the car at time t

The problem aims at engaging students with differentiation (and integration) routines in a setting where functional relationships are not given by formulas, meaning that the familiar analytic differentiation routines are not readily applicable. The problem also creates a potential for students to form connections between the objects of derivative and definite integral, through exploring the relationship between rate of change (RoC) and accumulation (Thompson & Silverman, 2008). In solving the main problem, we expected the students to sketch the velocity graph using either the geometrical interpretation of the derivative as the slope of a tangent line or a numerical approximation of the RoC of the position $s(t)$. For the follow-up problem, we expected the students to reconstruct the distance from the graph of the instantaneous velocity $s'(t)$ either by estimating the area under the curve or by using Riemann sums for the approximation of a definite integral.

The problem uses an everyday setting and everyday language, and contains one visual mediator, the position-time graph, providing a partial mathematization of the problem. The same holds for the follow-up problem, which is also built around a single graph. The questions are closed, but phrased so as not to suggest particular solution methods, thus providing students with the opportunity to engage with various mathematical routines, both familiar and less familiar. We deliberately chose a setting where the function was not given algebraically, to emphasise the need for numerical techniques for handling even very simple real-world problems (Kaput, 1994). To answer the first two questions, students first need to interpret the graph and extract the information needed, routines that should be well-known to them from their previous studies. They then need to use this information to first estimate the instantaneous velocity, which, as already mentioned, can be done either numerically, building on the notion of RoC and using the numerical differentiation routine introduced in the warm-up task, or geometrically, by estimating the slope of the graph at the indicated times. Whichever method the students choose, it will require them to engage with routines less familiar to them. Second, they are asked to estimate average velocity, a routine familiar to them from upper secondary mathematics and physics.

The task allows students to reflect on the relationship between average velocity and instantaneous velocity, in terms of the position-time graph. Since we expect both numerical and geometrical routines to be used for estimating the velocity, there is opportunity for discussion about the relation between these two methods. This resonates with the observation made by Hauger (2000) that students often make sense of instantaneous RoC in terms of average RoC. In the last part of the problem, students then need to build on their work on the first problem in order to find the information needed to engage in a graph construction routine (Viirman & Nardi, 2021), namely constructing the velocity-time graph, another topic where student difficulty is well documented (e.g. Berry & Nyman, 2003). Numerically estimating the instantaneous velocity at several points and plotting them against time, students can see the functional character of the relationship, even though it is not given by a formula.

The follow-up problem allows students to engage in numerical integration routines, discussed in the next section. The problem also allows students to see how the Fundamental Theorem of Calculus connects RoC and accumulation (Thompson & Silverman, 2008), by realising that the accumulated position, that is, the distance, at time t is given by the integral of its RoC, that is, the velocity, from the starting point to t. Doing this for different values of t, the students can gain a sense of the integral as an accumulation function of the quantity whose RoC we know. This is something that students often struggle with, since they are not used to thinking of the upper limit as varying (ibid.). The problem also requires students to interpret the negative area under the velocity curve in terms of accumulated position, something that research has shown to be a challenge for students (Bressoud et al, 2016). For further detail on the “Svenssons’ vacation” problem, see (Viirman & Pettersson, 2019).

The “Vase” problem

For this problem, each group of students were given a glass vase, and were asked to estimate its volume. The main aim of the problem was having the students engage in integration routines in an everyday setting where no mathematization of the problem was given. The lecture preceding the session had dealt with, among other topics, solids of revolution, and it was expected, although not explicitly stated, that the students would use this idea to consider the vase as generated through rotation around a central axis, thus in essence reducing it to a two-dimensional problem. Like the “Svenssons’ vacation” problem, this problem is formulated completely in everyday language, but it is also open in the sense that it does not suggest any particular solution strategy. Additionally, the formulation contains a quite concrete physical object – the vase itself. The mathematization of the problem then requires the students to construct their own visual mediators representing this concrete object by projecting it onto a planar surface. Since we ourselves found producing a reasonably accurate projection of the contour of the vase in two dimensions time-consuming, we had prepared a handout (Figure 3a) to make available to the students if needed. Indeed, all groups turned out to need the handout. The problem as such does not require the students to formulate any mathematical narratives, since it only asks for a numerical estimate of the volume of the vase. However, we of course expected the students to be able to formulate mathematical narratives describing the method they had used to come up with the estimate. As for mathematical routines, the openness of the problem allows for several solution strategies, allowing students to make use of both routines established earlier in their studies, and routines introduced during the course. However, similarly to the "Svenssons' vacation" problem, we intentionally designed the problem in such a way that no explicit formula was available. In addition to emphasising the need for numerical techniques (Kaput, 1994), this also led the students away from relying on the standard algorithmic integration routines familiar to them from previous studies. For many students, integration is synonymous with finding the anti-derivative, inserting the endpoints of the interval and subtracting, and working with numerical integration can help support a broader understanding of the definite integral as the limit of a sum (Blomhøj & Hoff Kjeldsen, 2007).

We expected the numerical estimation routines employed to be of one of two main types: either the students would make use of some form of geometrical estimation routine, trying to approximate the area by geometric means, or they would use a Riemann sums routine of the kind previously discussed during the course. The former routine would be familiar to them already from upper secondary school, whereas the latter, although not new to them, could be expected to be less firmly established. Letting students work with Riemann sums in this kind of numerical setting has a further advantage in that, as pointed out by Thompson and Silverman (2008), the role of the variable of integration is often a mystery to students, but by considering Riemann sums this role can be made clearer. When students have to decide how to divide the interval of integration, the integration variable becomes experientially real to them.

EMPIRICAL OBSERVATIONS

We report from the first two iterations of the intervention[1], building on field notes, made by the authors during the problem-solving seminars run by them, and on students' written productions. A general observation was that only a small number of students, approximately 15[2] out of 120, chose to participate in the first trial. An explanation given by students who chose not to participate was that the IO sessions would deprive them of the usual tutorials, seen as important for the examination. As this was a pilot, the limited participation was perhaps not entirely a bad thing, but it was still somewhat disappointing, particularly since those few students who chose to participate reported that they mostly found the sessions helpful for their understanding and providing variation in a quite demanding mathematics course. Hence, for the second iteration of the course, we made the problem-solving sessions a regular part of the work of all four seminar groups. In this way, we could avoid the sense that the activities were optional and thus of less relevance.

As for the problems we designed, some worked better than others. The first session, on limits, turned out to be too demanding for the students, who were not able to make any progress without support from the teacher. As we realized, the main difficulty stemmed from the fact that in the lectures, the notion of limit had been introduced for functions rather than for sequences, making the basic premise of the problem difficult for the

[1] Since both authors changed affiliations during or after the second iteration, we have no insight into the later iterations.
[2] Precise numbers varied between the different problem-solving sessions.

students to understand. Indeed, in contrast to the situation in, for instance, France, in Sweden limits of sequences do not appear at all in school calculus, and play a less significant role in introductory university calculus (Viirman, Vivier & Monaghan, under review). For the second run of the course, we made further modifications and simplifications to the problem, but it still worked quite poorly. Hence, it was decided to replace it for the third iteration. The "Chinese whispers" graph sketching problem in the third session worked reasonably well in both iterations, but the students' work on it has not yet been analyzed in any detail.

Students' work on the "Svenssons' vacation" problem

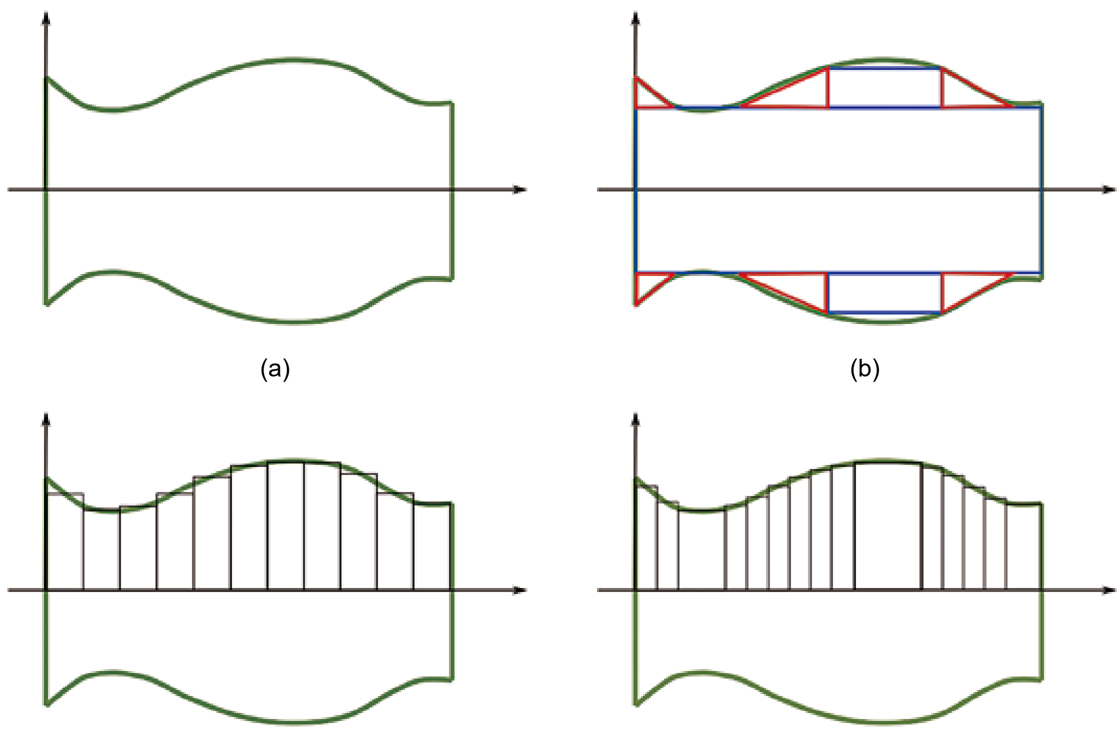
The session on derivatives, on the other hand, worked very well. The formulations of the problems were not as abstract as for the session on limits, and to our satisfaction, the students employed a variety of different routines for solving the tasks. The introductory problem introduced a technique for numerical differentiation, and indeed some of the students used this for dealing with the main problem. However, some students chose to adapt previously established routines for graphically estimating derivatives to the situation at hand. This was in accordance with our aims and expectations in designing the problem, where we wanted students to have the opportunity to employ previously establishes routines as well as to engage in "thoughtful imitation" (Sfard, 2008, p. 251), adapting the recently learned numerical differentiation routine to the task at hand. Further observations made included, for instance, difficulties with interpreting and plotting negative velocities, with some students choosing to plot them as positive (cf Berry & Nyman, 2003). Interestingly, one group of students anticipated the follow-up problem, saying that it should be possible to get the distance out of the graph for the velocity by computing the area under the curve. That is, at least to some students the connection between numerical differentiation and integration suggested itself. An insightful comment was made by one student: "It feels like the graph should be smooth, but I do not know why. All functions in physics are smooth". This prompted a quite useful discussion concerning the behaviour of acceleration and deceleration.

Somewhat surprisingly, when the problems were used for the second iteration, there was less variety in approaches taken. All groups (at least in the sessions run by the authors) used the technique introduced in the introductory problem. Why this was the case we can only speculate, but it is possible that the students participating in the first iteration, having volunteered to participate, were perhaps more interested in problem solving, and thus more willing to try different methods. There might also have been subtle differences in how the problems were introduced, causing the students in the second session to view them as more closely connected.

Students' work on the "Vase" problem

For this problem, the student groups were handed one physical copy of the vase each and were asked to estimate its volume. Once they had an estimate they were satisfied with, they were to report to the teacher, who wrote the estimated value on the blackboard. As new groups reported their estimates, in some cases the faster groups decided to reconsider their work and revise their estimates. When all the groups had finished working, we let the students check their estimates by filling the vase with water from the sink that was available at the back of the classroom.

Since the formulation of the problem gave no indication of the choice of method, the routines employed were more diverse than for the "Svenssons' vacation" problem. All groups treated it as a case of a solid of revolution, and as expected they struggled with constructing a plane projection of the vase, causing us to provide them with the handout we had prepared. Once the groups had this handout, the methods they employed for estimating its area, and thus the volume of the vase, differed considerably. As previously mentioned, we had expected two main avenues of approach, and we did indeed see examples of both in both iterations. Taking the session led by the first author in the first iteration as an example, among the groups employing geometrical approximation routines, one tried approximating the vase by a cylinder, drawing a straight horizontal line across the outline of the vase in such a way that the areas created between the outline and the straight line were approximately equal. This is a type of approximation routine familiar to the students already from secondary school, where it is used to calculate the area of polygons. The second group approximated the shape of the vase by a sequence of straight lines, thus creating a polygonal shape whose area could easily be calculated (Figure 3b), again a routine familiar from previous studies. Both these approaches yielded reasonable, but not particularly precise, estimates of the volume. The third and the fourth group, on the other hand, used Riemann sum routines, dividing the interval into subintervals, treating the curve as constant on these intervals, and summing up the areas of the resulting rectangles. The third group used a large number (about 25) of subintervals of equal length, while the fourth group made use of the fact that the slope of the shape of the vase differed along its height to use fewer intervals of unequal length. The students in these two groups discussed actively what height for each rectangle they should use in order to get a better approximation, which is a step towards different formulas for numerical integration. A third approach



(a) (b) (c) (d)

Figure 3: Examples of strategies for estimating the volume of the vase.

was taken by the last group who, somewhat to our surprise, chose to employ least-squares approximation in order to find an explicit function describing the shape of the vase, which they could then integrate using familiar integration routines in order to find an estimate of the volume. The students in this group took a quite fine partition of the interval and then used fourth- and fifth-order polynomials to approximate the shape. It turned out that this approximation was the best one. Least squares approximation had been discussed in the previous linear algebra course, yet another example of how previously established routines were employed in new contexts. We were particularly glad to see that the students applied the least-squares technique, since this technique is widely used in other areas and is a standard method to fit functions to a set of data.

CONCLUDING REMARKS

Despite the difficulties with the session on limits, the first iteration of the intervention worked well enough to warrant a full-scale implementation for the second iteration. In particular the problems on differentiation and integration conformed well with our intentions, confronting the students with the need for numerical techniques, and allowing them to engage in a variety of routines, well-established as well as more recently learnt. Judging from conversations with students, and from course evaluations, the intervention was well-received, although so far, we have not made any systematic efforts to evaluate to what extent the problem-solving sessions have contributed to student learning.

Apart from the need to redesign some of the problems, one important lesson we bring to the planning of future iterations is that more care needs to be taken when orchestrating the sessions, to get students to actively discuss their work with other groups and sharing their solutions. In the sessions that worked well, we managed to get students engaged in problem solving, but the kind of collaborative atmosphere where students can learn from each other's work did not quite develop. Most successful in this regard was the session on integration, possibly (and perhaps somewhat paradoxically) because of the element of competition involved. Still, we need to think further on how to enable students to benefit from the solution strategies of others, without the sessions becoming too time-consuming, or deteriorating into "show-and-tell" (Stein, Engle, Smith & Hughes, 2008). One option might be to use peer-review during the sessions, having the groups provide critique of each other. It is our hope that future iterations of the project will include further steps in this direction.

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