SOMETIMES MATHEMATICS IS DIFFERENT IN ELECTRICAL ENGINEERING

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Abstract

In this contribution we will present an ongoing research project on mathematical practices in electrical engineering. Starting with interesting phenomena we have encountered in our research regarding the relationship of mathematics and engineering, we provide some general thoughts on the notions application and modelling. We then present our own vantage point: Using the Anthropological Theory of the Didactic (ATD), we take an institutional point of view on mathematical practices. This allows us to conceptualise two ideal type mathematical discourses corresponding to different epistemological constitutions of mathematical knowledge in mathematics courses for engineers and in advanced courses in electrical engineering, respectively. We will enrich our presentation with short vignettes of our latest research results to illustrate the kind of insights that the institutional point of view enables us to gain particularly regarding educational issues.

Key words: Anthropological theory of the didactic, mathematical practices, electrical engineering, application and modelling

INTRODUCTION

The study of engineering mathematical practices is an important topic in engineering mathematics education (Alpers, 2020; Winsløw et al., 2018). Explicitly focusing on the specific content related needs of engineering mathematics for didactic analyses enables a deeper understanding of practices and potential learning difficulties related to them. A deep analysis of teaching materials and students' works can then also open up new ideas for teaching design. In an ongoing research project on mathematical practices in Signal Theory, we refer to the Anthropological Theory of the Didactic (ATD) (see Bosch et al., 2019; Chevallard, 1992; Chevallard et al., 2022) and, besides other, its understanding of praxeology to model mathematical practices in electrical engineering to address those issues. Other recent ATD related studies on mathematical practices in engineering are done by Bartolomé et al. (2019), Florensa et al. (2018), González-Martín (2022), Palencia (2022), Rønning (2021) and Schmidt and Winsløw (2021). In our research project we developed three foci: First, with a focus on subject specific mathematical practices, we introduced an extended praxeological model to reconstruct the mathematical discourse that justifies the mathematical practices in signal theory (Peters & Hochmuth, 2021). In (Hochmuth & Peters, 2021) we show, how students' solutions to a signal theory exercise can be analysed and understood on the basis of our previous analyses. The second focus

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considers the epistemological relationship between mathematics and electrical engineering (Hochmuth & Peters, 2020; 2022). We showed that epistemological relations between mathematics and engineering can be important for a detailed description and analysis of mathematical practices. Considering epistemological aspects of mathematical practices within the framework of the ATD makes those aspects accessible for didactical analyses and design. The third focus highlights the potential of our ATD analyses for teaching design (Peters, 2022). Here the emphasis is on possible connections between mathematics as taught in higher mathematics courses and mathematics in engineering courses. Based on previous work, we develop an idea for teaching design to foster such connections without the need for the introduction of application examples or the complete restructuring of the course. In an early phase of our research project, when we analysed teaching materials and students' works and had mainly the first focus in mind, we came across two interesting phenomena: First, we repeatedly encountered a deficit-orientation towards mathematical practices of engineers when discussing data and corresponding intermediate analysis results with colleagues. The mathematical practices we studied generally did not follow socio-mathematical norms of academic mathematics[1]. From the standpoint of academic mathematics, mathematical practices in electrical engineering courses like Signal and System Theory (SST) seemed to be sometimes wrong, incomplete and sketchy. This was ascribed to limitations in engineering studies, but was nevertheless seen as a (necessary) deficit. Second, from the perspective of academic mathematics some engineering mathematical practices were difficult or impossible to understand. For example, in our analyses, we could identify arithmetic transformations but could not explain their significance and reasons from the standpoint of academic mathematics (cf. our analysis vignette).

Both phenomena seemed also to be connected. Engineering mathematical practices, that were difficult or impossible to understand, were often simply framed as deficits from the mathematicians' point of view. As fruitful as analyses are that reveal such differences, it is not satisfactory to interpret them sweepingly as deficits. The question is also which deficits from the perspective of academic mathematics are part of an adequate electrotechnical mathematical practice and which are not? Which of the identified deviations from academic mathematics hinder the learning of mathematical concepts in engineering and which deviations are necessary for the adequate teaching and learning of engineering mathematical practices? However, the same question also arises for non-deficit mathematical practices in engineering. Is any access to a mathematical concept that is adequate from an academic mathematical point of view also beneficial for the learning of mathematical practices of engineers? We found these questions important specifically for our analyses, but also relevant to engineering mathematics education in general. Only a detachment from the deficit-oriented view of engineering mathematics practices makes these questions accessible. We could see that those mathematical practices were pragmatic and also necessary to solve specific epistemological problems related to physics or engineering, respectively. In our studies, we later, when we developed the second focus, were able to relate some apparent mathematical shortcomings of engineering mathematics to epistemological issues, which cannot be clarified by inner-mathematical considerations alone and which sometimes underlie

[1] We speak of academic mathematics when we refer to mathematical research institutes and institutes of mathematics at universities. We distinguish academic mathematics from other mathematical institutions, such as mathematics courses for engineers that are part of engineering study programs. In our studies we also use the ATD concept of didactical transposition to connect the relationship of different mathematical institutions with our analyses (cf. Peters, 2022).

the conceptualisation of mathematical knowledge in electrical engineering (Hochmuth & Peters, 2020; 2022). We realised that those positive aspects of the engineering way of doing mathematics were difficult to acknowledge and analyse from the perspective of academic mathematics alone.

In this contribution we want to proceed from this and move on to some general reflections on the concepts of *application* and *modelling*, which are often used in studies of engineering mathematics education to capture the relationship of mathematics and engineering. Thereby, the epistemology of this relationship generally remains implicit and unquestioned. On the other hand, applications of mathematics and modelling problems are often present as important design aspects to improve the teaching of mathematics to engineers (e.g. Alpers, 2020). After bringing up some critique on the standard concepts of application and modelling, we present our stance which enables us to avoid some of the difficulties inherent in application and modelling. After introducing central concepts of ATD and illustrating our stance with two vignettes from previous work, we come back to the notions of application and modelling and show further possibilities of alternative conceptualisations from the viewpoint of ATD.

ON THE CONCEPTS OF APPLICATION AND MODELLING AND AN ALTERNATIVE VANTAGE POINT TO BETTER UNDERSTAND ENGINEERING MATHEMATICAL PRACTICES

There are various definitions and understandings of application and modelling (e.g. Blum et al., 2007), most of which separate between an extra-mathematical world and mathematics. Often application of mathematics and mathematical modelling are seen as related to each other. Niss, Blum, and Galbraith (2007) summarises this relationship and their understanding as follows

> During the last one or two decades the term '*applications and modelling*' has been increasingly used to denote all kinds of relationships whatsoever between the real world and mathematics. The term 'modelling', on the one hand, tends to focus on the direction 'reality → mathematics' and, on the other hand and more generally, emphasises the *processes* involved. Simply put, with *modelling* we are standing outside mathematics looking in: 'Where can I find some mathematics to help me with this problem?' In contrast, the term 'application', on the one hand, tends to focus on the opposite direction 'mathematics → reality' and, more generally, emphasises the *objects* involved - in particular those parts of the real world which are (made) accessible to a mathematical treatment and to which corresponding mathematical models already exist. Again simply put, with *applications* we are standing inside mathematics looking out: 'Where can I use this particular piece of mathematical knowledge?' (p. 10f)

These widely held understandings of the relationship of mathematics and engineering can be very fruitful, especially in teaching design. But it is also regularly noted that the realisation and implementation in everyday teaching is problematic. Barquero et al. (2013) give a survey of literature illustrating the difficulties and barriers as a general problem for the dissemination of modelling activities. They use their own projects to identify and categorise difficulties and barriers. Besides other, they

> focus on describing some constraints related, in the first place, to what may be called the *dominant*

> *epistemology*, that is, the way our society, the university as an institution and, more particularly, the community of university teachers and students, understand what mathematics is and what its relation is to natural sciences. (p. 316)

Regarding the meaning of applications Barquero et al. (2013) reconstructed an epistemology of *applicationism* in the relationship of mathematics to other sciences and identify it as a restriction on the notion of mathematical modelling:

> One of the main characteristics of applicationism is that it greatly restricts the notion of *mathematical modelling*. Under its influence, modelling activity is understood and identified as a mere application of previously constructed mathematical knowledge or, in the extreme, as a simple exemplification of mathematical tools in some extra-mathematical context artificially build in advance to fit these tools. (p. 317)

Regarding the two worlds of mathematics and "the rest of natural sciences" they note that "it is furthermore supposed that both 'worlds' evolve with independent logic and without too many interactions" (p. 318). Also, they note that "in general, the mathematics taught present a highly stereotyped and crystallized structure that does not mingle with the systems that are modelled and, moreover, the mathematics taught are never 'modified' as a consequence of being applied." (p. 319). We can now ask whether an unquestioned academic mathematical perspective on mathematical practices in engineering, where mathematics is also seen as "never modified", can be linked to applicationism? From the applicationsm point of view, it is suggestive to understand mathematical practices in engineering only as more or less deficient applications of previously constructed academic mathematical knowledge.

Regarding the notion of mathematical modelling Bissell and Dillon (2000) note

> Mathematical modelling forms an important part of engineering education and practice. Yet precisely what is meant by the term 'modelling' is often extremely unclear - and, moreover, much of what students are told about the subject is considerably problematic from both a philosophical and a pedagogical point of view. (p. 3)

In their study they look at mathematical modelling from the practicing engineer's perspective. From this perspective the usual modelling cycles are too simplistic to capture mathematical activities of engineers. Also, they note that instead of creating mathematical models in engineering, it is much more important for practicing engineers to use already existing mathematical models (Bissell & Dillon, 2000, p. 4). This shift of perspective on mathematical modelling in engineering, enables an understanding of mathematical models without a necessary separation of mathematics and the rest of the world: here engineering is not only the context for application or the source of the modelling problem. Engineering itself is already mathematised. They also characterise necessary skills for using models: *manipulation* is "the ability to modify the form of the basic model, using algebraic and other skills; essentially 'mechanical'", *interpretation* is the "ability to interpret the modified form of the model in a way relevant to the situation; essentially 'reactive'", and application is the "ability to apply the interpretation and make appropriate recommendations; essentially 'proactive'" (p. 4). Note that they speak of applying the interpretation with respect to the relevance of the situation. Here the applied mathematics does not necessarily remain unchanged as it is the case in applicationism. By connecting their considerations about mathematical modelling also with the general question of "the position of mathematics in engineering" they state that "there is clearly a significant difference between what a mathematician calls ‘doing mathematics’ and what an engineer calls ‘doing mathematics’.” (p. 6).

In our research project we found that to better understand these different ways of doing mathematics and to analyse mathematical practices in engineering without restrictions to applicationism or a deficit-oriented view, we needed a different vantage point: A vantage point that enables us to understand the engineering specific justifications and explanations of mathematical practices and allows for deeper content specific analyses than the considerations by Bissell and Dillon. The ATD enables this, among other things, through the principle of *institutional dependence of knowledge*: In different subject specific institutional contexts mathematical practices are justified, substantiated, validated and constituted differently than in academic mathematics. Also, Castela (2015) emphasises[2] the advantages of an institutional approach to research on mathematical knowledge in different contexts that is fundamental to the ATD. This approach “provides a powerful tool to investigate the mathematics dimension of human social activities in any context, without referring to academic mathematics.” (Castela, 2015, p. 18). This can contribute to counteracting a deficit-oriented view of mathematical practices and provide a deeper understanding of mathematical practices in other siences.

INSTITUTIONAL POINT OF VIEW AND MATHEMATICAL DISCOURSES

Building on Castela’s work, we have introduced a specific extended[3] praxeological model (Peters & Hochmuth, 2021) that allows us to analyse mathematical practices in electrical engineering, particularly taking into account the engineering-specific institutional conceptualisation of mathematical practices. We also showed how institutional analyses of engineering mathematical practices can be related to and help understand individual students’ solutions to exercises (Hochmuth & Peters, 2021). In the following we will present vignettes from both studies to illustrate our approach.

Alongside the institutional dependence of knowledge, *praxeology* is another ATD concept that is important for our work. In ATD a praxeology is a basic epistemological model to describe institutional knowledge in the form of two inseparable, interrelated blocks: the praxis block (know-how) consists of types of problems or *tasks* (T) and a set of relevant *techniques* (τ) used to solve them. The logos block (know-why) consists of a two-levelled reasoning discourse. On the first level, the *technology* (θ) describes, justifies and explains the techniques and on the second level the *theory* (Θ) organises, supports and explains the technology. In short praxeologies are denoted by the 4T-Model $[T,\tau,\theta,\Theta]$.

We illustrate how the concept of praxeology, especially our extended praxeological model, is able to produce (in the sense of a phenomenotechnique, cf. Bosch et. al., 2019) an understanding of the engineering-specific conceptualisation of mathematical practices. We will introduce our extension to the praxeological model in the course of the following exemplary analysis at the step, where the need for an extension concretely arises. For this, we consider an exercise of an SST course at a German university that is taught in the second year

[2] She addresses the relationship of academic mathematics and mathematics in vocational contexts.

[3] With respect to the standard praxeological model of ATD. In our extension we differentiated techniques and technologies according to two mathematical discourses, see below.

of an electrical engineering study program. First, we focus on the lecturer's sample solution, i.e. the taught knowledge in SST. The context of this exercise is amplitude modulation. The exercise under consideration is[4]:

Graphically display $x(t) = A\cos(2\pi f_0 t) + \frac{Am}{2}\cos(2\pi f_0 t + \Omega t) + \frac{Am}{2}\cos(2\pi f_0 t - \Omega t)$ in the complex plane as a rotating phasor with varying amplitude using the relationship $\cos(2\pi f t) = \Re\{\exp(j2\pi f t)\}$.

The lecturer sample solution is:

One first writes

$$x(t) = A\cos(2\pi f_0 t) + \frac{Am}{2}\cos(2\pi f_0 t + \Omega t) + \frac{Am}{2}\cos(2\pi f_0 t - \Omega t) \tag{1}$$

$$= A\,\Re\{\exp(j2\pi f_0 t)\} + \frac{Am}{2}\Re\{\exp(j(2\pi f_0 t + \Omega t))\} + \frac{Am}{2}\Re\{\exp(j(2\pi f_0 t - \Omega t))\} \tag{2}$$

$$= \Re\left\{ \exp(j2\pi f_0 t)\underbrace{\left[A + \frac{Am}{2}\exp(j\Omega t) + \frac{Am}{2}\exp(-j\Omega t)\right]}_{A(t)} \right\} \tag{3}$$

and interprets the expression in the square bracket as a real-valued time-dependent amplitude *A(t)*, which modulates the carrier phasor $\exp(j2\pi f_0 t)$ rotating at frequency f_0 in Figure 1.

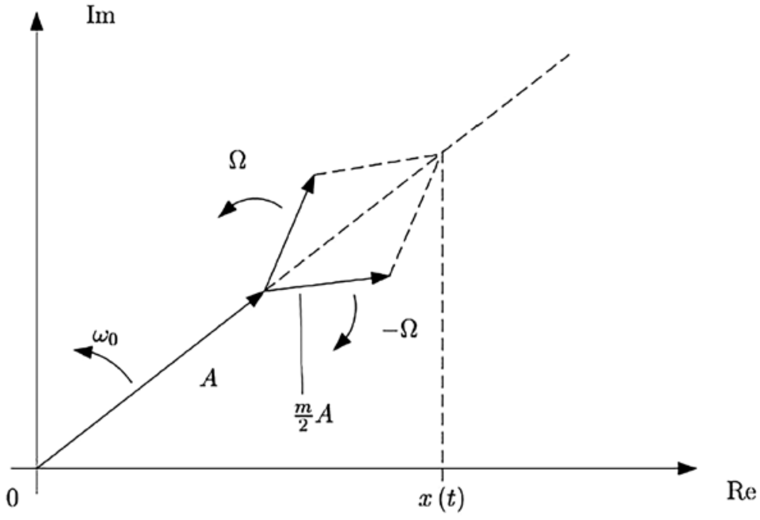

Figure 1: Representation of $x(t)=A[1+m\cos(\Omega t)]\cos(2\pi f_0 t)$ as the real part of a rotating phasor $A(t)\exp(j2\pi f_0 t)$ with $\omega_0=2\pi f_0$.

The sample solution of the exercise represents institutional knowledge of signal and system theory. We now can assign the praxeological components to the steps in the solution. Since this is one exercise, the reconstruction of types of tasks is not relevant here. In the full analysis, as presented in (maybe this is an

[4] Literal translation from the German exercise sheet by the author. The number of the figure is adjusted to the figure counter in this publication.

inconvenient pagebrake 2021) and (Hochmuth & Peters, 2021), we use the method of considering subtasks to further structure the analysis. For this analysis vignette, we focus on techniques as part of the praxis-part and on technologies as part of the logos-part of praxeologies. Tasks and theory will not be further considered and will therefore be understood as SST-tasks and SST-theory, i.e. tasks relevant in the institution SST and theory as the second level of the praxeological reasoning discourse according to the institution SST.

From line (1) to line (2), in the sample solution, using the relationship $\cos(2\pi ft) = \Re\{\exp(j2\pi ft)\}$ is a *technique* τ. Here, connections between the representations of a complex number in polar form and in exponential form are relevant for the justification, the *technology* θ . Both, technique and technology, for this step are known by the students from Higher Mathematics courses earlier in the study program. Here we can see, that for this SST-exercise techniques and technologies from Higher Mathematics courses are relevant. The institutional knowledge of signal and system theory therefore has connections to knowledge from a different institution.

Before we go deeper into details here, we come back to the already introduced idea of institutional dependence of knowledge to clarify what this means for our analysis in particular. Following (Castela, 2015), an institution is

> a stable social organisation that offers a framework in which some different groups of people carry out different groups of activities. These activities are subjected to a set of constraints, - rules, norms, rituals - which specifies the institutional expectations towards the individuals intending to act within the institution I. […] Institutions tend to constrain their subjects but conversely they provide the resources (material and cultural) necessary for activities to take place. (p. 7)

Institutional conditions, norms and aims constitute the technological-theoretical discourse and the practices available. This means that different types of tasks are relevant in different institutions, different solution techniques are adequate, different reasoning discourses are acceptable, and different reasons to study a subject occur. Thus, if one focuses on a specific mathematical subject in different institutions, different praxeologies could emerge. In the following we will show that in the context of our research, the institutional knowledge in SST shows references to other relevant institutions and corresponding institutional mathematical discourses[5]. Our analysis so far showed a praxeology concerning techniques and a technological discourse of dealing with complex numbers that can be assigned to an institution Higher Mathematics (HM). We denote this praxeology by $[T, \tau_{HM}, \theta_{HM}, \Theta]$. In our work, we used the textbook by Strampp (2012), students' lecture notes, and exercises from a course on Higher Mathematics for Engineers based on this textbook to characterise the mathematical knowledge associated with this institution, i.e. the HM-discourse: It is characterised by an internal mathematical conception without concrete references to reality, an orientation towards a generalising rational of academic mathematics, a concentration on calculation rules, and the inclusion of school mathematics concepts. The reason why complex numbers are studied in HM-courses is because they are useful for solving polynomial equations and they are important objects of calculation. Arrows in the Gauß-diagram are used to graphically illustrate calculation rules and properties.

[5] The term discourse refers to the logos part of praxeologies: In ATD, logos is considered as a discourse on praxis (reasoning discourse), but since praxis and logos are dialectically interrelated, every aspect of praxis (i.e. tasks or techniques) is also related to the institutional discourse. Reasoning discourses are institutionally dependent, and so are the respective techniques and technologies. The notion of institutional discourse enables us to differentiate analytically between techniques and technologies that could be associated to different institutional discourses respectively.

When we now look at the solution step from line (2) to line (3) also techniques from the HM-discourse occur: The real parts of the summands are factored out, calculation rules for the exponential function are applied, and the resulting common factor $\exp(j2\pi f_0 t)$ is factored out. But it is difficult to understand the reasons for this transformation from the standpoint of Higher Mathematics. Why make a clearly structured expression more complicated? Also, from the way complex numbers are taught in the HM-course, drawing three phasors associated each with one of the summands in line (2), seems much more obvious than drawing phasors associated to the more complicated expression in line (3). To understand why this transformation is carried out, we have to look for the engineering reasoning that is not part of the HM-course: $x(t)$ is transformed in a specific way to graphically represent principles of amplitude modulation, that could not be represented by a graphical representation of line (1) or (2) (see also our second vignette of an analysis of a student solution to this exercise below). The cosine representation in line (1) does not allow to separate the different frequencies or angular velocities of the carrier-signal, $\omega_0 = 2\pi f_0$, and the message signal, Ω . This is, however, the core of both the representation in line (3) and the graphical representation in Figure 1 in the sample solution. There is no justification within our reconstructed HM-discourse, that gives the reason for the step from line (2) to line (3). So, the technological discourse underlying the step from line (2) to line (3) differs from the HM-discourse. This other mathematical discourse belongs to a different institution. In our analyses we denoted this other mathematical discourse as an electrotechnical mathematics-discourse (ET). In our work, we most notably use studies by Bissell and Dillon (Bissell & Dillon, 2000; Bissell, 2004; 2012) and the electrical engineering textbook by Albach (2011) to characterise this mathematical ET-discourse. In contrast to the HM-discourse, the ET-discourse has references to reality. The degree to which this reference to reality is made explicit can vary greatly, along with a different degree of formalisation and abstraction. In addition, it is characterised by a "linguistic shift" (Bissell & Dillon, 2000, p. 10) in the way of talking about mathematics and mathematical practices and an electrotechnical-typical way of "system-thinking". One reason why complex numbers are studied according to the ET-discourse is that they allow oscillating signals to be described algebraically in a very suitable way and visualised graphically as phasors. This visualisation does not serve to illustrate calculation rules or properties of complex numbers but represent important analysis tools, e.g. for AC circuits (cf. Albach, 2011) or amplitude modulation. For a more comprehensive description of the discourses see (Hochmuth & Peters, 2021; Peters & Hochmuth, 2021; Peters, 2022).

The step from line (2) to line (3) can be associated to a praxeology $[T, \tau_{HM}, \theta_{ET}, \Theta]$. In the next step, the expression in line (3) has to be interpreted in order to draw the Gauß-diagram, cf. Figure 1. The part denoted by $A(t)$ must be interpreted as a modulation process (τ_{ET}). This is justyfied because the phasor which varies in length with $A(t)$ represents a general periodic signal (θ_{ET}). In this praxeology technique and technology are from the mathematical ET-discourse, $[T, \tau_{ET}, \theta_{ET}, \Theta]$.

This brief illustrative insight into our analysis of the sample solution shows that the solution of this task can be linked to different praxeological configurations ($[T, \tau_{HM}, \theta_{HM}, \Theta]$, $[T, \tau_{HM}, \theta_{ET}, \Theta]$, and $[T, \tau_{ET}, \theta_{ET}, \Theta]$) drawing on the two different institutional mathematical discourses. We could observe that both institutional discourses are interrelated and show up in different combinations of techniques and technologies. Transitions or shifts between the two mathematical discourses constitute epistemological ruptures in the sense that they each follow a different rational. These ruptures often remain implicit, although they represent important aspects. They indicate places that are not accessible from a single mathematical discourse and its techniques, and thus mark something additional to be learned. Neither the modelling and application-point of view nor the standard praxeological model are sufficient for this kind of analyses: Application and modelling both entail a conceptualisation of electrical engineering knowledge as consisting of inner-mathematically justified mathematical practices[6] and extra-mathematical engineering knowledge[7]. The praxeology $[T, \tau_{HM}, \theta_{HM}, \Theta]$ could be interpreted as a purely inner-mathematical, as the HM-discourse has strong relations to academic mathematics and no references to reality. So this part of the analysis could be related to the application- or modelling view. But there is no accompanying extra-mathematical engineering knowledge where this praxeology (i.e. the knowledge that is modelled within ATD with this praxeology) is applied to. The mixed praxeology $[T, \tau_{HM}, \theta_{ET}, \Theta]$, where a technique from the HM-discourse gets a new ET-discourse meaning, does not fit this viewpoint at all. The standard praxeological model that would allow to take different institutional origins of practices into account and to describe the knowledge in form of HM- and ET-praxeologies does not allow to shed light on the interrelatedness of the two mathematical discourses.

As a second vignette we present a short analysis of a student solution to this exercise[8] from (Hochmuth & Peters, 2021). This is an example of a solution, where the necessary shifts between the two mathematical discourses do not occur (cf. Figure 2).

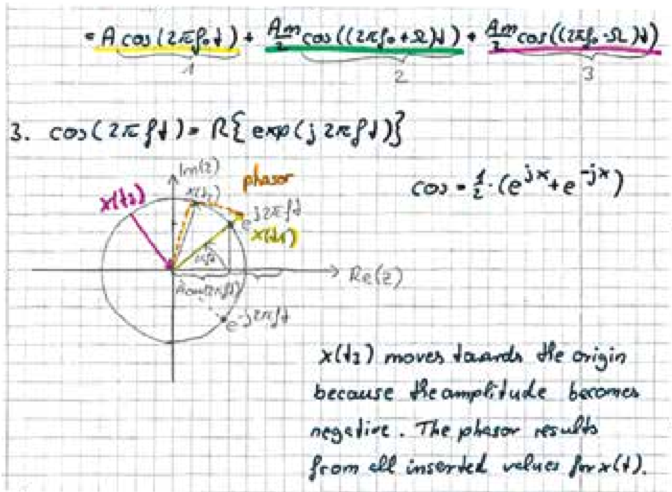

Figure 2 A student solution to the exercise (Hochmuth & Peters, 2021)

At the top we see each of the three cosine-terms separately underlined and each given a number. Each term is thus interpreted individually as something to be drawn. These numbers can also be found in the diagram; the respective phasors are marked accordingly ($x(t_1)$, $x(t_2)$, and $x(t_3)$). While underlining mathematical terms is a technique neither specific to the HM-discourse nor to the ET-discourse the idea represented in this technique, that each term is something to be drawn individually, is a technology of the HM-discourse (θ_{HM}): Each cosine-term stands for a complex number that could be drawn as an arrow starting at the origin of the Gauß-diagram. The sum of three complex numbers then could be drawn as the geometric sum of the

[6] Learnt in mathematical courses and applied later in different contexts.

[7] Providing the context for the application of the mathematical knowledge or the modelling problem.

[8] To protect the student's privacy, we have rewritten the student solution. We omitted the assistant's marking.

respective arrows. The student tried to graphically add the three arrows (dashed line in the diagram in Figure 2). Additionally, the diagram also contains elementary properties of complex numbers: the connection between cosine and the complex exponential function and the complex conjugate. This student solution reproduces the HM-discourse by drawing a diagram similar to diagrams from the mathematics service course where the Gauß-diagram and the unit circle are used to illustrate properties of complex numbers (τ_{HM}). Aspects indicating a connection to amplitude modulation are missing and transitions to the ET-discourse do not occur. This student solution does not produce a diagram that is capable of illustrating aspects of amplitude modulation.

SOMETIMES MATHEMATICS IS DIFFERENT: A DISCUSSION

This short analysis vignettes show the relevance of taking into account the specific mathematical ET-discourse for a deeper understanding of mathematical practices of electrical engineers. Reconstructed mathematical practices from a lecturer sample solution of a signal and system theory-task contain aspects of both discourses, the HM-discourse and the mathematical ET-discourse. An analysis vignette of a student solution to this exercise showed that referring only to the HM-discourse is both a possible student action and not sufficient to solve the task.

We already argued that the standard conceptualisations of the relationship of engineering and mathematics, modelling and application, are not able to capture the complex nature of engineering mathematical practices in this way. From our ATD perspective, we see this relationship not as a relation between independent fields of knowledge. At the core of our approach is the acknowledgement of mathematical practices of engineers as institutional mathematical practices in their own right and with engineering specific conceptualisations of mathematical knowledge. Relations to academic mathematics are present, e.g. in our analysis in the HM-discourse. Also, relations to mathematics developed within the engineering institutions are present, e.g. in our analysis in the ET-discourse. In (Peters, 2022), these relations are discussed in more detail. The mathematical discourses interact in complex ways and are not understandable from the standard modelling and application point of view.

Nevertheless, there are conceptualisations, that are connectable to our stance. Concerning an understanding of application we would like to mention the work by Schmidt and Winsløw (2021). Using an analysis of didactical transpositions between institutions Mathematics and Engineering, they develop a method to design Authentic Problems from Engineering (APE). In their approach the idea of applying mathematical knowledge to engineering starts with engineering. From there they look for possibilities to let the engineering knowledge interact with the mathematical concepts. This approach is specifically capable of counteracting the problem of applicationism. Concerning modelling, we already mentioned the perspective of Bissell and Dillon (2000) who shift the focus to the *use* of mathematical models and show how in the historical process engineers developed mathematical practices specific to their needs and aims (see also Bissell, 2004; 2012). Another important line of development is the reformulation of modelling from an ATD perspective as it is presented for example can we change this to: by Garcia et al. (2006). This reformulation seems particularly appropriate to us here, of course, because we share the same framework of ATD. But apart from that, we also consider the approach fruitful because it takes a decidedly epistemological and institutional perspective on modelling. A first important interpretation of modelling within the ATD is "that modelling is [not] just one more aspect or dimension of mathematics, but that mathematical activity is essentially a modelling activity in itself" (p. 232). Two statements are then important. First mathematical modelling is not restricted to "'mathematization' of non-mathematical issues" (p. 232). Also, inner-mathematical activities are understandable as modelling activities. Second, they highlight the meaning of modelling activity from the standpoint of ATD:

> In the framework of the ATD, what is relevant is not the specific problem situation proposed to be solved (except in 'life or death' situations), but what can be done with the solution obtained –that is, with the constructed praxeology–. The only interesting problems are those that can be reproduced and developed into wider and more complex types of problems. The study of those *fertile problems* provokes the necessity of building new techniques and new technologies to explain these techniques. In other words, the research should focus on those *crucial questions* that can give rise to a rich and wide set of mathematical organizations. Sometimes, those *crucial questions* have an extra-mathematical origin, sometimes they have not. (p. 233)

They summarise the proposed understanding of the modelling process as

> a process of reconstruction and interconnection of praxeologies of increasing complexity (*specific → local → regional*). This process should emerge from an initial question that constitutes the rationale of the sequence of praxeologies. From this questioning, some *crucial questions* to be answered by the *community of study* should arise. (p. 233)

Within ATD this approach was further developed under the notion of study and research paths (SRP) (e.g. Bartolomé et al., 2019; Chevallard, 2006; Florensa et al., 2018). From our research perspective especially, the focus on a crucial question that guides the research or learning process is relevant here. This question is not only a question from a specific context but a question that also has the potential for questioning the content specific institutional rationales. In our analysis, in the context of complex numbers, two different rationales, each within a specific institutional mathematical discourse, occurred. We can connect those rationales with the idea of different ways of doing mathematics from Bissell and Dillon (2000) and shed more light on the meaning of the "significant difference between what a mathematician calls 'doing mathematics' and what an engineer calls 'doing mathematics'." (p. 6).

So, sometimes mathematics is different but the question appearing now from our perspective is: when is which discourse relevant? Our analyses show that both discourses show up in electrical engineering exercise solutions: The HM-discourse with its orientation towards academic mathematics is important, as well as the engineering specific mathematical ET-discourse. To be able to successfully solve exercises as our example, students have to know when which discourse is adequate and when to switch. However, this switching itself is often not made explicit in teaching. Analyses of student solutions of this exercise, like the one above, show that students difficulties could be connected to this question of switching between mathematical discourses (Hochmuth & Peters, 2021). Our analyses can therefore help to identify hurdles for students and to achieve subject-related clarifications. Lecturers can use this to explicitly address difficulties when discussing sample solutions.

In addition, the acknowledgement of mathematical practices of electrical engineers as a justified institutional discourse in itself can prevent a deficit-oriented view and open up new possibilities for teaching, task design and student guidance. In our analyses, for example, we refer to different reasons for studying complex numbers. In the Higher Mathematics course, complex numbers are important because they allow to solve every polynomial equation. In electrical engineering, the general problem of the solvability of polynomial equations is not the main interest. Here, complex numbers are particularly important because they allow to describe oscillating signals. Such reasons to study a mathematical concept are part of the logos block, thus especially part of the institutional mathematical discourses, and may not be explicitly addressed in teaching. So, students may implicitly learn one reason to study complex numbers in one course and a different reason in other courses. The different reasons may then not fit together or even contradict each other. This could convey the impression that mathematics as taught in Higher Mathematics courses is not useful for or disconnected from engineering. In (Peters, 2022) we present a concrete teaching development idea for mathematics service courses based on our research findings. In doing so, we illustrate how the difference between the two identified mathematical discourses can be used constructively in teaching development.

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