ON ONLINE TEACHING AND LEARNING OF MATHEMATICS: WHAT FUTURE RESEARCH CAN BE EXPECTED BY MATHEMATICS EDUCATION RESEARCH?

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Abstract

The current COVID-19 pandemic has made us rethink ways to provide educational opportunities. The purpose of this discussion paper is to identify issues and challenges in online teaching and learning of mathematics to be explored in future research, and what mathematics education can contribute to this research area. I first identified recent relevant research themes from 2015 to 2020 by reviewing key literature from mathematics education journals. International mathematics education research studies have been examining various issues around designing online learning environments, social interactions and communication in online teaching and learning, and the role of resources and tools. These concerns also exist among Japanese teachers, and I argue Lesson Study can be used creatively in order to seek effective ways to deliver online teaching and learning of mathematics.

Key words: COVID-19 pandemic, Online teaching of mathematics, Teachers' concerns, Ideas for online teaching, Online lesson study

1. INTRODUCTION

"It is absolutely evident that we should be prepared that a bad year [a year when natural disasters cause massive problems in our society] will come back in future, and in good years we need to make our life ready [for such coming disasters]. But it is such a usual thing that we completely forget about this in good years [bad years with natural disasters]. Some people might say we can enjoy our life while we are forgetting such disasters, but I leave if this is a good attitude or not to our personal philosophy of life. However, I strongly consider at least people who are involved in national politics and administrations must not forget to treat this 'amnesia' (for coming disaster)." (Torahiko Terada, 1934)

This statement was written by Japanese physicist Torahiko Terada (1878-1935) in 1934. Although this essay is mainly concerned with natural disasters such as earthquake, typhoon etc. and the context was Japan's national defence in the time when the world situation was very unstable due to the great depression at that time, this statement seems to be very relevant the situation which we are currently facing a world-wide pandemic caused by the COVID-19 virus. He also pointed out human society is a systemic organisation with complex structures, and even a damage in a part of the organisation can be fatal for functions.

As in Terada's statement, we indeed did not prepare for the current uncertain situation in 'good years'

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while we were 'enjoying' our usual life before the pandemic, but what can we do in this situation to support pupils' mathematics learning? One answer is to adopt online teaching and learning of mathematics. While we acknowledge there are many different types and forms for online teaching and learning, in this paper online teaching and learning are defined as educational activities entirely or partially with internet (Ko and Rossen, 2017), in particular:

- Synchronous, real-time teaching and learning with video conferencing tools such as Skype, Zoom, etc.
- Asynchronous teaching with use of recorded video, voices etc.
- Use of structured learning with web-sites, online quizzes, PDF, etc.

In the case of Japan, however, in April 2020 synchronous, real-time online teaching has been provided by only 5% of the 25,223 schools which are currently closed, while some kind of digital resources have been used by only 29% (Japanese Ministry of Education and Sport). Uegatani et al. (2021) reported "During the pandemic, the majority of public primary and secondary school children in Japan did not have mathematics instruction for a few months, except for homework through self-learning." (p. 2). Through deeply analysing interview data from the two students, thier study also found these students lost opportunities in being members of their communities, and missed social interactions in mathematics problem solving. With other Japanese researchers, I have conducted a survey in teachers' mental readiness to try online teaching in April 2020. The survey was distributed through email and Social networking sites, and in total 207 teachers in elementary and junior high school teachers returned their answers. In this survey, it was suggested that these teachers have a positive attitude towards the use of online teaching and learning of mathematics, they are anxious about how to actually make their teaching interactive and how to deal with unexpected technical issues. In early 2021 we are still facing rather unstable situations with COVID-19, and it is worth exploring what known and future research issues are in the online teaching and learning of mathematics.

Reflecting on such experience since 2020, the purpose of this discussion paper is to identify issues and challenges in online teaching and learning of mathematics from international research studies, and consider what mathematics education research can be conducted in future. In particular, I work with the following research questions: "What research studies in online teaching and learning of mathematics have been conducted in mathematics education research between 2015-20, and what future research studies are expected to be conducted?".

These questions are first explored by a systematic literature search, and by reviewing some key literature identified by the review. Grant et al (2009) suggest there are at least 14 types of review, and identifies one as an 'Overview' – a summary of the literature that attempts to survey the literature and describe its characteristics (p. 94). The potential strength of this approach is, "Overviews can provide a broad and often comprehensive summation of a topic area and, as such, have value for those coming to a subject for the first time." (p. 99), and I will take this approach in this paper.

In what follows, first, I have conducted a literature search by using Web of Science in order to gain an overview of existing knowledge about online teaching and learning of mathematics, in particular what research has been done, with what topics, etc. A reason for using Web of Science is a) it is one of the trusted databases in research studies, b) to use CitNetExplorer which enables to visualise citation links between research papers based on Web of Science database (Van Eck and Waltman, 2014). Although I am aware that this has certain limitations (e.g. Web of Science does not include PME proceedings), the database includes

recent papers (between 2015-2020) published in the major mathematics education journals, i.e. *Educational Studies in Mathematics (ESM)*, *Journal for Research in Mathematics Education (JRME)*, *ZDM*, *Journal of Mathematical behavior (JMB)*, *Journal of Mathematics Teacher Education (JMTE)*, *Research in Mathematics Education (RME)*, *International Journal of Mathematics Education in Science and Technology (IJMEST)*, *International Journal of Science and Mathematics Education (IJSME)* and *International Journal for Technology in Mathematics Education (IJTME)*. Therefore, I consider the use of Web of Science and CitNetExplorer can provide a useful starting point to gain an overview of research studies between 2015-20. I will then explore what future research studies are expected to be conducted informed by the literature review described above, together with opinions expressed by 207 elementary and junior high school teachers in an open question, who participated in an online survey in April 2020.

2. LITERATURE IN ONLINE TEACHING IN MATHEMATICS EDUCATION JOURNALS IN 2015-2020

2.1. Overview of development

Drijver (2015) pointed out design, teachers and educational context are the important factors for successful integration of educational technologies in mathematics education. While these factors are certainly essential for online teaching and learning, it seems there are some other specific elements for online teaching and learning in addition to these three factors. In 2012, *ZDM* journal published a special issue on online mathematics education (Vol. 44(6)). According to Borba and Llinares (2012), this Special Issue explored the following two themes (p. 698):

- The impact of online collaboration on the constitution of networks, communities, and the issue of sustainability (the role of social networking)
- The impact of online collaboration on the learning and development of teachers

These themes were certainly important when Social Networking sites and online collaborative tools such as Google Docs were rapidly developing, and their educational potentials were recognised (e.g. Zhou et al., 2012). We can also see other themes such as multimodality, enhancement of student learning, pedagogical shifts, and online resources were also recognised as the major research themes at that time (ibid., p. 698).

From the 9 mathematics education journals, I initially extracted 75 papers which include the keyword 'Online', in February 2021. After screening with the criteria ('Is this about online teaching and learning of mathematics', and 'Does it include use of online resources, but not the mere use of online survey to collect data?'), 68 papers were initially identified (see Table 2 in Appendix). I then used CitNetExplorer and VOS viewer to visualise citation links as follows with my own notes (minimum citation score: 3).

From this list and visual networks created by CitNetExplorer (and VOS viewer), the following points might be observed:

- Fig. 1 suggests there are many themes which have been studied in this area of research, e.g. online assessment, e-learning, transforming classroom with internet, digital resources, teachers' professional development, etc.

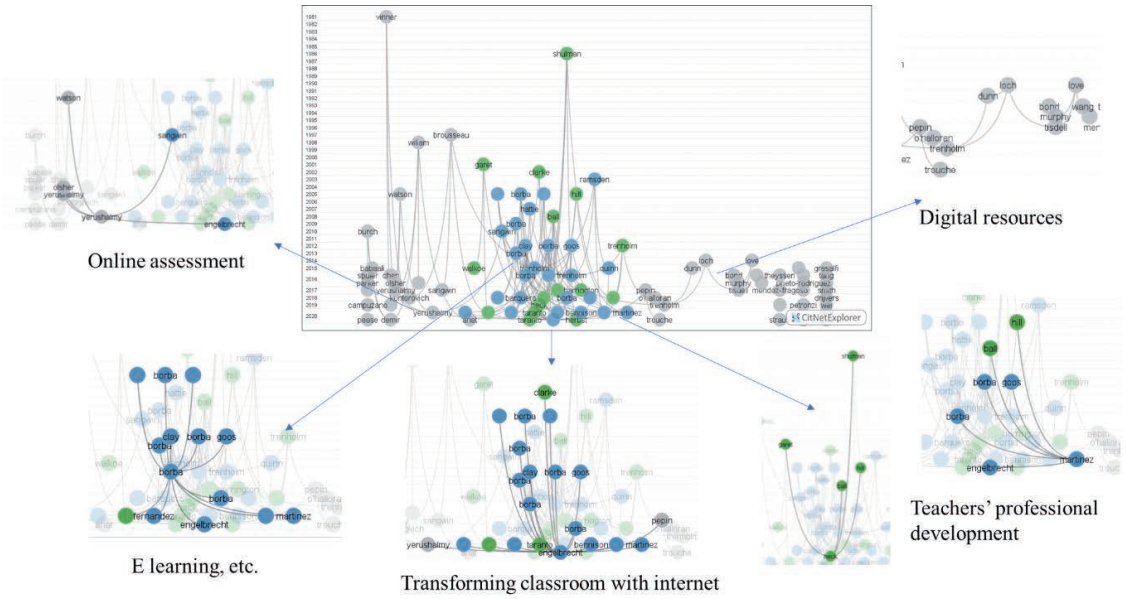

Figure 1. Visualisations of citation network in studies related to 'online' maths education in maths education journals in 2015-2020

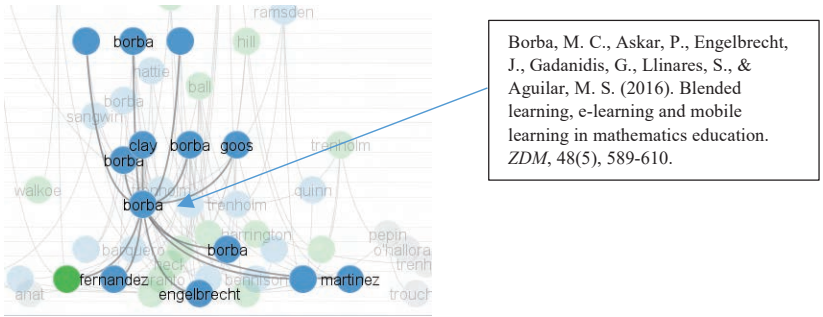

Figure 2. Citation network around Borba et al. (2016)

- Fig. 2 visualises a citation network around e-learning. The figure 2 left shows many papers are 'integrated' in Borba et al in 2016, published in *ZDM*. This paper is also one the most cited papers.
- We can see many papers which are related to online teaching and learning are published in *ZDM* journal, and indeed *ZDM* published a special issue 'Online mathematics education and e-learning' in 2020 (vol. 52(5)), which published 16 papers. Fig. 3 created by VOS viewer suggests that the paper by Engelbrecht et al. (2020) seems to summarise the existing knowledge very well.

Therefore, a useful starting point for grasping existing knowledge and insights in online teaching of mathematics might be by reviewing Borba et al. (2016) and papers from the *ZDM* special issue in 2020 (This does not of course mean we should not refer to papers published in the other journals and where appropriate I will refer to some).

2.2. What were discussed in Borba et al. (2016) and *ZDM* Special issue in 2020?

The paper Borba et al. (2016) seems to summarise many papers as well as being cited by various

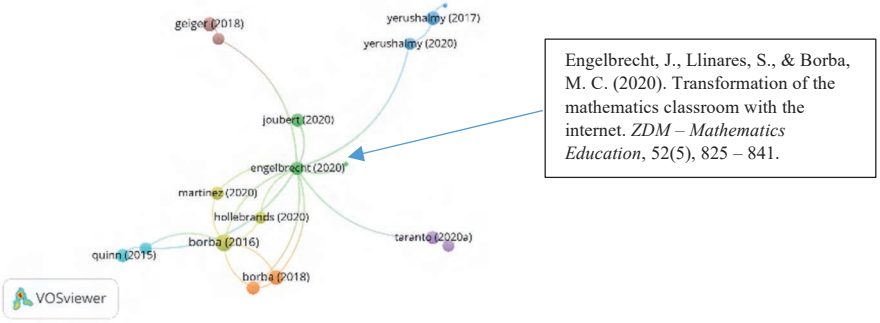

Figure 3. Citation network around Engelbrecht et al. (2020)

literature. This is one of the survey papers on research on mathematics education as one of the achievements of ICME13 conference (Hamburg, Germany, 2016), led by Borba, Kaiser and Kaur. In this paper, Borba et al. (2016) surveyed papers in major mathematics education journals such as *Educational Studies in Mathematics*, *Journal for Research in Mathematics Education*, *For the Learning of Mathematics*, *The Journal of Mathematical Behavior* etc. and some Latin American journals such as *BOLEMA: Boletim de Educação Matemática* and summarised on the studies related to blended learning, e-learning and mobile learning in mathematics education. They first acknowledged the rapid development from 2010 such as Web 2.0 technology , and identified the following five areas as important for research, i.e. the use of mobile technologies, MOOCs (massive open online courses), digital libraries and designing learning objects, collaborative learning using digital technology, and teacher training using blended learning (p. 590). This paper concludes the five issues to be recognised as areas to be further developed and explored in future research studies (p. 660), i.e. relationships between students and mathematical knowledge shaped by mobile technologies, the potential of MOOCs to affect access to and the quality of mathematics, how to organize and design online mathematics resources, design and use of online learning spaces for collaborations, and teachers use of blended learning.

In 2020, followed by Borba et al. (2016) and the five suggestions, *ZDM* published another special issue in Online mathematics education and e-learning. In a survey paper of this issue, Engelbrecht et al. (2020) stated that this Special Issue published papers around the following concerns, i.e. principles of design of MOOCs and professional development opportunities for teachers, social interactions and construction of mathematical knowledge, and tools and resources (p. 827).

While we always value face-to-face interactions, considering the current situations caused by COVID-19 pandemic, more online or blended professional development opportunities for teachers will be considered. Also, due to the development of online communication tools such as Zoom or Slack, it is easier to construct international network, but our understanding of design and processes, necessary factors for successful and sustainable developments and critical evaluations of various online/blended courses remain to be investigated. This topic is indeed explored in this Special Issue (Martinez et al., 2020; Hollebrands and Hollylynne, 2020; Goos, et al., 2020; Joubert, et al., 2020; Bennison, et al., 2020). Among many interesting studies, I found lesson studies with blended formats to be particularly interesting as this is a creative way of conducting a professional development for teachers. Conducting online lesson study is a relatively new idea (see Norwich,

et al., 2016 as one of the early attempts). Joubert et al. (2020) explored how a lesson study can be undertaken in a blended format to support isolated teachers across Southern Africa. They identified eleven aspects, i.e. technology; group; learning management system; online facilitation; technological pedagogical content knowledge (TPACK); (mobile) learning strategies; a lesson planning form; backward design; time; photos, videos and reports; and refection questions, which will be essential to achieve a blended lesson study with three dimensions (Collaborations, Instructional Development and Iterative Improvement Process) to support isolated teachers (p. 923).

Online communication and collaboration was already recognised as an issue needing to be tackled in 2005 (Engelbrecht and Harding, 2005) as well as Borba and Llinares (2012), and Borba et al. (2016), and it seems this is still one of the major concerns in the online teaching and learning of mathematics (also flipped classrooms, e.g. Murphy, et al. 2016). In Engelbrecht et al. (2020), for social interactions and construction of mathematical knowledge issues around theories, roles of social artefacts or tools such as online collaboration, mind mapping or sharing narratives in online forums in meaning constructions, collaboration spaces including facilitating asynchronous online discussion, and roles of media are discussed (p. 830). In the Special Issue, for example, Cendros-Araujo and Gadanidis (2020) explored how online collaborative mind mapping contributed to develop preservice teachers' knowledge in mathematics education. They found activities such as arranging items, sizing, highlighting, linking or separating ideas with collaborative mind maps can enrich their mathematics and pedagogical knowledge.

Finally, this Special Issue examines resources and tools. Technological tools might be seen as digital/nondigital artefacts for teaching and learning mathematics such as graphic calculators, dynamic geometry software, etc. (Monaghan et al., 2016; Fujita, 2018), and indeed they are one of the most important elements of the teaching and learning of mathematics. Engelbrecht et al. (2020) state "The development of new digital technologies provides new opportunities to mathematics educators, and new ways of thinking about how the teaching and the design of teaching–learning environments evolve, generating new practices and establishing goals which we did not think about several years ago" (p. 831), and this Special Issue contains studies around this theme. The study by Aguilar and Puga (2020) is particularly interesting. They explored how 30 Mexican engineering undergraduate students used information from the Internet to complete directed tasks in the definite integral. The study distinguishes the two types of help seeking, i.e. Executive help seeking ("situations in which student's purpose is to find something or someone to help them to solve a problem, or reach a goal on their behalf", p. 1006) and instrumental help seeking ("more focused on promoting a self-understanding of an idea or a problem-solving process", p. 1006). The results are somewhat expected, i.e. students' help-seeking attempts were dominated by instrumental help-seeking behaviours for procedural problems (e.g. calculating …) by using search engines and key words to identity useful sources. However, it seems students' help-seeking behaviours remain to be explored further, e.g. how such learning experience actually contributes to develop students' mathematical understanding and thinking, shape their understanding and thinking in particular ways, etc. as this kind of help-seeking from the Internet will be more and more common.

Another important idea is the Personal Learning Environments, defined as "systems that provide support to students so they can take control of their learning (by setting their own learning objectives) and manage their own learning content to achieve these learning objectives" (Borba, et al. 2016, p. 601), or *hyper-personalisation* (Engelbrecht et al., 2020). For example, depending on availabilities of devices, a learner

might use just a laptop or desktop computer for online learning, others use only a tablet/mobile phone, or combining both computer and tablet/mobile phone in their learning. The 'one-size-fit-all' approach cannot be adopted for building effective learning environments, but it should be carefully organised to support not only students' development of mathematical knowledge and understanding, self-regulated learning which is considered as a key element of online learning (Dabbagh and Kitsantas, 2012).

3. WHAT COULD MATHEMATICS EDUCATION RESEARCH CONTRIBUTE TO THE ONLINE TEACHING AND LEARNING OF MATHEMATICS?

3.1. Japanese teachers' concerns

Based on Borba et al. (2016) and *ZDM* Special issue in 2020, design online learning environments, social interactions and communication in online teaching and learning, and roles of resources and tools are recognised as particularly important. In this section I shall discuss what future research studies are expected to be conducted. Before proposing some ideas, I refer to some views expressed by Japanese teachers when they faced the first school closure in March-April 2020. Although these views were taken from Japanese teachers, they should be interesting and relevant to mathematics education researchers and teachers internationally.

In April 2020, a survey was conducted investigating teachers' mental readiness to use online teaching and learning of mathematics to in total 207 elementary (N=105) and junior high school (N=102) teachers. The survey asks how teachers were mentally ready to use online teaching for mathematics (1-5 scale), together with 22 perception questions such as their attitude towards online teaching (e.g. how much they are interested in online teaching for mathematics), social norms (e.g. how their colleagues think about online teaching), resources (e.g. how they feel about technological difficulties when using online teaching) and so on. A quantitative analysis of this survey suggests that Japanese teachers have relatively positive attitudes towards the use of online teaching and learning of mathematics. However, their sense of crisis is very high and they are anxious about how to actually make their teaching interactive and how to deal with unexpected technical issues. In one of the open questions, these participants were asked to write their comments or thoughts on situations they were facing. When we read these comments, it seems that concerns expressed by the teachers (who participated in the survey) are roughly classified into the following themes (Table 1).

These concerns expressed by Japanese teachers also echo what international research studies have been focusing on, which is discussed in the previous section (Borba et al., 2016; Engelbrecht et al., 2020 and more), i.e. rethinking and designing curriculum contents for online teaching by considering students' help-seeking behaviour from the internet, social interactions and collaborations in online teaching, organising network environments and tools, and professional development. Almanthari et al. (2020) also reported that Indonesian secondary school mathematics teachers (N=159) felt various birrias such lack of teachers' knowledge, not having internet connections, the difficulties to understand the content, lack of access to a computer etc., exist to implement e-learning effectively.

We can also see the teachers expressed various thoughts and feelings including more general educational issues than mathematics, such as caring for students' mental health. Also, it seems that Japanese teachers are

Table 1 Concerns for online teaching expressed by Japanese teachers

Themes	Examples comments from open questions
Covering required mathematical contents	• I am worried the quality of learning will not be good enough (due to the school closure). (Junior high school) • Considering to catch up with the last year's maths topics which we have not finished, I feel it will be really difficult to cover all the required mathematical contents. (Junior high school) • The required mathematical contents is too much and this made both children and teachers anxious. (Junior high school) • I honestly am worried if I can finish teaching all the required maths contents in the Course of Study. (Junior high school)
Attainment including gaps	• There is a limit for elementary school pupils to undertake learning by themselves. How can we support this? (Elementary school) • Attainment decline due to the school closure (Elementary school) • I feel that gaps in students who can work by themselves or not will be visible in their attainment gaps (Junior high school) • Lessons will be reduced, and I am worried how students who are slower to understand can cope with. (Junior high school)
Caring students' mental health	• In March when I visited to my students while my school was closed, I felt there are many children who are just playing games, unhappy with their home environment, having no contacts with their friends, etc. and I think caring for their mental health is necessary in addition to supporting their academic learning. (Elementary school) • Even pupils who felt not to rather go to schools actually started saying they want to see their friends and attend lessons. I really consider it is essential for them to meet face-to-face in one place. (Junior high school)
Organising network and tools	• It is an urgent matter to establish online teaching, and provide learning opportunities to pupils. Because the school is closed, we now have time to organising environments. It is time to stop saying 'we cannot do this' but all local organisation and schools should at least try, and we just learn and cumulate 'know-how' for education in the next generations. (Elementary school) • I feel the network environments at home are not ready, it would be really difficult to practice online teaching for all the family. (Junior high school)
Expectations for changes	• Completely unpredictable things are happening, so I want to undertake what we can do with ICT thoroughly. We can sufficiently connect with our pupils via ICT tools. (Elementary school) • I actually expect online teaching will provide an opportunity to develop pupils' various competencies (Junior high school)
Interactions in teaching	• We are facing difficult situations in communicating each other as we cannot use pair or group activities. Therefore, I feel it has been difficult to deepen our understanding through interactions. I consider if tablets will be distributed to all the pupils, then this problem might be overcome. (Elementary school). • I now recognise it is really important for both teachers and pupils to learn interactively in one place. (Junior high school)

concerned if they can cover all the required contents specified by the Japanese Course of Study.

3.2. The use of Lessons Study for tackling challenges

Then, the question remains how can mathematics education research contribute to these issues in future research in online teaching and learning, in order to gain critical insights not only useful for online teaching

and learning but also for international mathematics education research (Stable and reliable internet connections and appropriate devices will be essential to achieve effective online teaching and learning. In the following suggestions, I assume this condition is satisfied, although this equity issue has been problematic, including soft equity issue described as "the lack of appropriation of necessary knowledge and positive self-identification" (Uegatani et al., 2021, p. 1)).

One way to tackle these issue is to use Lesson study, such as Collaborative Lesson Research described as "As a form of lesson study, CLR is an investigation undertaken by a group of educators, usually teachers, using live lessons to answer shared questions about teaching and learning" (Takahashi and McDougal, 2016, p. 519). Attention can be paid to the issue of how these tools and tasks can be used in online teaching to maintain social interactions and collaborations between teachers and pupils in mathematics in order to deepen their thinking and understanding of mathematical knowledge and concepts. Also, we still do not know, what online communication tools can be used with learning activities designed with GeoGebra (https://www.geogebra.org/), Desmos (https://www.desmos.com/), or CODAP (https://codap.concord.org/), or to what extent the use of online synchronous tools such as https://whiteboard.fi/ can replace face-to-face teaching in mathematics. For example, Fig. 4 is taken from my own use of whiteboard.fi in a session for UK trainee teachers who are exploring ANNA number patterns with Zoom, and how their thinking and understanding might be shaped in such teaching?

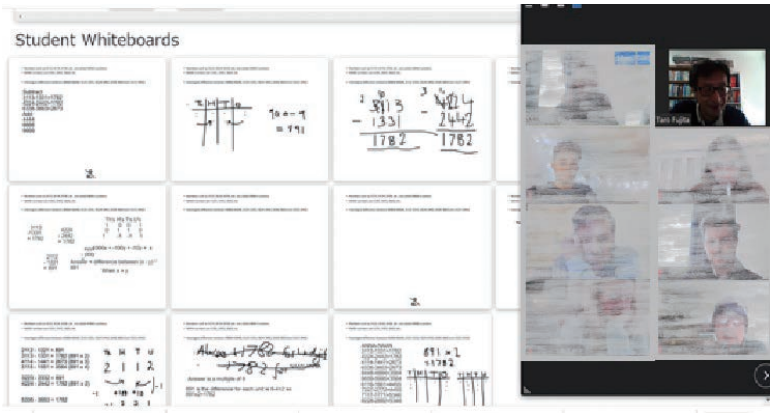

Figure 4. Use of whiteboard.fi in online teaching

Distributing appropriate devises for accessing online teaching is also important. We saw that one teacher commented that "I consider if tablets will be distributed to all the pupils, then this problem (interactions with pupils) might be overcome", and recent Japanese GIGA school initiative might be seen as a positive direction. However, is just distributing individual tablets enough? For example, in 2008, the Portuguese Government launched the 'e.escolinha' programme, which aimed to provide a laptop "Magalhães" to children in primary schools for the use at schools and homes, and by 2011, 500,000 children received their laptop. While there were some positive impacts on children's digital literacy skills, various social factors in education was not carefully considered by the government. In general, it was reported that without dialogues between policy makers and school practice, together with a lack of clear pedagogical guidance, the use of laptops in classroom was not more effective than 'traditional' lessons (Pereira, Pereira, et al., 2015). Here,

the idea Personal Learning Environment (PLE) might be a useful starting point. Little is known about how to organise students' PLEs for online learning of mathematics for Japanese pupils, including encouraging self-regulated learning (Dabbagh and Kitsantas, 2012). In relation to PLEs, I would like to introduce another idea 'a Personal Teaching Environment (PTE)' for teachers, which can be defined, by adopting Borba et al. (2016), as "systems that provide support to TEACHERS so they can take control of their TEACHING (by setting their TEACHING objectives) and manage their TEACHING content to achieve these TEACHING objectives". For example, a teacher might just use a computer to do online teaching, but other teachers might combine a computer with hand-written recognition devices in order to realise interactive, synchronous online teaching. Again, Lesson study can be used to critically examine how PLEs and PTEs might be organised in order to teach mathematics effectively, or supposing PLEs/PTEs are established to teach algebraic content, then these PLEs/PTEs should be reorganised differently when geometry is taught.

Also, we have seen in the literature, internationally how Lesson study is conducted in online/blended formats are becoming an interest in mathematics education research (e.g. Joubert, et al., 2020). Questions can be asked, for example, what online tools can be used to facilitate collaborative lesson planning, how video materials be used as prompts for post-lesson discussions and evaluations including asynchronous posts in online forums, etc., which has been actually happening since 2020 (e.g., in Japan, an online meeting hosted by Kumamoto University attached elementary school was held on February 20th 2021. Lessons were uploaded on YouTube and Zoom was used for meetings: https://elem.educ.kumamoto-u.ac.jp/2021_2_19-20_kenkyuhappyokai-3/ accessed in June 2021; in UK, Lesson study UK group developed a guidance for conducting online lesson study: https://lessonstudy.co.uk/2021/02/new-free-to-download-online-research-lesson-study-dos-and-donts/, accessed in June 2021).

4. CONCLUSION

In 2021, Japanese schools are currently open and many of mathematics educators are thinking we do not have to practice online teaching now or even in future. However, the COVID-19 pandemic really demonstrates us unexpected things can really happen, and we cannot assume our traditional face-to-face teaching will continue forever. Also other countries, such as in the UK, schools are forced to close, online teaching is happening, and teachers and pupils in schools are working hard to adopt this new ways of teaching and learning. Reflecting on the current situation, this discussion paper explored the questions "What research studies in online teaching and learning of mathematics have been conducted in mathematics education research between 2015-20, and what future research studies are expected to be conducted?" As we have seen in section 2, international mathematics education research studies (identified from Web of Science and organised by CitNetExplorer and VOS viewer) have been examining various issues around design online learning environments, social interactions and communication in online teaching and learning, and roles of resources and tools. In section 3, by reflecting on Japanese teachers' views on online teaching, I argue that mathematics education research (both international and Japanese) will be able to contribute to these issues by using Lesson Study creatively.

Of course, I am not saying undertaking only these issues will be enough, and there are many other

important research areas such as online assessment (e.g. Sangwin and Jones, 2017), which have not sufficiently been discussed in this discussion paper. Another concern seems to be attainment gaps which might be created by online learning. We simply do not know, if gaps between pupils' mathematical attainments will be wider or not in online teaching compared to face-to-face teaching, but this cannot be addressed with only using Lesson Study, and a longitudinal research study will be necessary in future.

As we saw in Terada's statement at the beginning of this paper, perhaps at this time we need to develop more robust pedagogies for online teaching to prepare for the next crisis which will come in future. Indeed, as one of the Japanese teachers stated in the open question in the survey "It is time to stop saying 'we cannot do this' but all local organisations and schools should at least try, and we just learn and cumulate 'know-how' for education in the next generations". This attitude is necessary for mathematics education research to overcome our challenges caused by COVID-19 pandemic.

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Appendix

Table 2 Extracted papers from Web of Science

Authors	Title	Journal	Year
Heck, DJ; Plumley, CL; Stylianou, DA; Smith, AA; Moffett, G	Scaling up innovative learning in mathematics: exploring the effect of different professional development approaches on teacher knowledge, beliefs, and instructional practice	ESM	2019
Petronzi, D; Staples, P; Sheffield, D; Hunt, TE; Fitton-Wilde, S	Further development of the Children's Mathematics Anxiety Scale UK (CMAS-UK) for ages 4-7years	ESM	2019
Borba, MC; Chiari, ASD; de Almeida, HRFL	Interactions in virtual learning environments: new roles for digital technology	ESM	2018
Kontorovich, I	Why Johnny struggles when familiar concepts are taken to a new mathematical domain: towards a polysemous approach	ESM	2018
Sangwin, CJ; Jones, I	Asymmetry in student achievement on multiple-choice and constructed-response items in reversible mathematics processes	ESM	2017
Demir, M; Souldatos, I	Exploring Students' Online Homework Completion Behaviors	IJTME	2020
Rosa, P; Petraskova, V	Potential of Maple as a tool for improving financial education of future teachers	IJTME	2017
Mendez-Fragoso, R; Villavicencio-Torres, M; Martinez-Moreno, J	Design And Use Online Platforms To Learn Mathematics And The Use Of Them In Simulations Of Problems In Applied Sciences	IJTME	2017
Bond, V	Using Online Professional Learning Communities to Encourage Dialogue in University/College Mathematics	IJTME	2016
Martin, CS; Polly, D; Wang, C; Lambert, RG; Pugalee, DK	Perspectives and Practices of Elementary Teachers Using an Internet-Based Formative Assessment Tool: The Case of Assessing Mathematics Concepts	IJTME	2016
Prieto-Rodriguez, E	It Just Takes so Much Time! A Study of Teachers' Use of ICT to Convey Relevance of Mathematical Content	IJTME	2016
Anat, K; Einav, K; Shirley, R	Development of mathematics trainee teachers' knowledge while creating a MOOC	IJMEST	2020
Campuzano, JCP; Roberts, AP; Matthews, KE; Wegener, MJ; Kenny, EP; McIntyre, TJ	Dynamic visualization of line integrals of vector fields: a didactic proposal	IJMEST	2019
Maxwell, M; Gleason, J	Item efficiency: an item response theory parameter with applications for improving the reliability of mathematics assessment	IJMEST	2019
Trenholm, S; Hajek, B; Robinson, CL; Chinnappan, M; Albrecht, A; Ashman, H	Investigating undergraduate mathematics learners' cognitive engagement with recorded lecture videos	IJMEST	2019
Howard, E; Meehan, M; Parnell, A	Live lectures or online videos: students' resource choices in a first-year university mathematics module	IJMEST	2018
Sangwin, CJ; O'Toole, C	Computer programming in the UK undergraduate mathematics curriculum	IJMEST	2017
Smith, RC; Shin, D; Kim, S	Prospective and current secondary mathematics teachers' criteria for evaluating mathematical cognitive technologies	IJMEST	2017
Tisdell, C; Loch, B	How useful are closed captions for learningmathematics via online video?	IJMEST	2017
Ozcan, ZC	The relationship between mathematical problem-solving skills and self-regulated learning through homework behaviours, motivation, and metacognition	IJMEST	2016
Parker, KA	A modularized tablet-based approach to preparation for remedial mathematics	IJMEST	2016

Authors	Title	Journal	Year
Murphy, J; Chang, JM; Suaray, K	Student performance and attitudes in a collaborative and flipped linear algebra course	IJMEST	2016
Reid, J; Wilkes, J	Developing and applying quantitative skills maps for STEM curricula, with a focus on different modes of learning	IJMEST	2016
Kensington-Miller, B; Novak, J; Evans, T	Just do it: flipped lecture, determinants and debate	IJMEST	2016
Trenholm, S; Alcock, L; Robinson, C	An investigation of assessment and feedback practices in fully asynchronous online undergraduate mathematics courses	IJMEST	2015
Quinn, D; Albrecht, A; Webby, B; White, K	Learning from experience: the realities of developing mathematics courses for an online engineering programme	IJMEST	2015
Hoadley, S; Tickle, L; Wood, LN; Kyng, T	Threshold concepts in finance: conceptualizing the curriculum	IJMEST	2015
Babaali, P; Gonzalez, L	A quantitative analysis of the relationship between an online homework system and student achievement in pre-calculus	IJMEST	2015
Dunn, PK; McDonald, C; Loch, B	StatsCasts: screencasts for complementing lectures in statistics classes	IJMEST	2015
Choi, A; Hand, B	Students' Construct and Critique of Claims and Evidence Through Online Asynchronous Discussion Combined with In-Class Discussion	IJSME	2020
Straub, MCP	A Study of Student Responses to Participation in Online Citizen Science Projects	IJSME	2020
Redmond, P; Gutke, H	STEMming the Flow: Supporting Females in STEM	IJSME	2020
Zhong, BH; Xia, LY	A Systematic Review on Exploring the Potential of Educational Robotics in Mathematics Education	IJSME	2020
Wei, B; Chen, ST; Chen, B	An Investigation of Sources of Science Teachers' Practical Knowledge of Teaching with Practical Work	IJSME	2019
Balta, N	High School Teachers' Understanding of Blackbody Radiation	IJSME	2018
Theyssen, H; Struzyna, S; Mylott, E; Widenhorn, R	Online Physics Lab Exercises-a Binational Study on the Transfer of Teaching Resources	IJSME	2016
Wang, JR; Chen, SF	Development and Validation of an Online Dynamic Assessment for Raising Students' Comprehension of Science Text	IJSME	2016
Tang, KY; Wang, CY; Chang, HY; Chen, SF; Lo, HC; Tsai, CC	The Intellectual Structure of Metacognitive Scaffolding in Science Education: A Co-citation Network Analysis	IJSME	2016
Chen, CH; Chiu, CH	Collaboration Scripts for Enhancing Metacognitive Self-regulation and Mathematics Literacy	IJSME	2016
Trenholm, S; Alcock, L; Robinson, C	The Instructor Experience of Fully Online Tertiary Mathematics: A Challenge and an Opportunity	JRME	2016
Luz, Y; Yerushalmy, M	Students' conceptions through the lens of a dynamic online geometry assessment platform	JRME	2019
Harrington, RA; Burton, L; Beaver, C	Answering the call by developing an online elementary mathematics specialist program	JRME	2017
Hjalmarson, MA	Learning to teach mathematics specialists in a synchronous online course: a self-study	JRME	2017
Walkoe, J	Exploring teacher noticing of student algebraic thinking in a video club	JRME	2015
Herbst, P; Ko, I; Milewski, A	A heuristic approach to assess change in mathematical knowledge for teaching geometry after a practice-based professional learning intervention	RME	2020
Drijvers, P	Tools and taxonomies: a response to Hoyles	RME	2018

Authors	Title	Journal	Year
Erixon, EL	Learning activities and discourses in mathematics teachers' synchronous oral communication online	RME	2016
Taranto, E; Arzarello, F	Math MOOC UniTo: an Italian project on MOOCs for mathematics teacher education, and the development of a new theoretical framework	ZDM	2020
Araujo, RC; Gadanidis, G	Online collaborative mind mapping in a mathematics teacher education program: a study on student interaction and knowledge construction	ZDM	2020
Yerushalmy, M; Olsher, S	Online assessment of students' reasoning when solving example-eliciting tasks: using conjunction and disjunction to increase the power of examples	ZDM	2020
Pease, A; Martin, U; Tanswell, FS; Aberdein, A	Using crowdsourced mathematics to understand mathematical practice	ZDM	2020
Taranto, E; Robutti, O; Arzarello, F	Learning within MOOCs for mathematics teacher education	ZDM	2020
Engelbrecht, J; Llinares, S; Borba, MC	Transformation of the mathematics classroom with the internet	ZDM	2020
Trouche, L; Rocha, K; Gueudet, G; Pepin, B	Transition to digital resources as a critical process in teachers' trajectories: the case of Anna's documentation work	ZDM	2020
Joubert, J; Callaghan, R; Engelbrecht, J	Lesson study in a blended approach to support isolated teachers in teaching with technology	ZDM	2020
Quinn, D; Aarao, J	Blended learning in first year engineering mathematics	ZDM	2020
Martinez, S; Guinez, F; Zamora, R; Bustos, S; Rodriguez, B	On the instructional model of a blended learning program for developing mathematical knowledge for teaching	ZDM	2020
Fernandez, C; Llinares, S; Rojas, Y	Prospective mathematics teachers' development of noticing in an online teacher education program	ZDM	2020
Hollebrands, KF; Lee, HS	Effective design of massive open online courses for mathematics teachers to support their professional learning	ZDM	2020
Bennison, A; Goos, M; Geiger, V	Utilising a research-informed instructional design approach to develop an online resource to support teacher professional learning on embedding numeracy across the curriculum	ZDM	2020
O'Halloran, KL; Beezer, RA; Farmer, DW	A new generation of mathematics textbook research and development	ZDM	2018
Barquero, B; Bosch, M; Romo, A	Mathematical modelling in teacher education: dealing with institutional constraints	ZDM	2018
Galleguillos, J; Borba, MD	Expansive movements in the development of mathematical modeling: analysis from an Activity Theory perspective	ZDM	2018
Geiger, V; Mulligan, J; Date-Huxtable, L; Ahlip, R; Jones, DH; May, EJ; Rylands, L; Wright, I	An interdisciplinary approach to designing online learning: fostering pre-service mathematics teachers' capabilities in mathematical modelling	ZDM	2018
Yerushalmy, M; Nagari-Haddif, G; Olsher, S	Design of tasks for online assessment that supports understanding of students' conceptions	ZDM	2017
Borba, MC; Askar, P; Engelbrecht, J; Gadanidis, G; Llinares, S; Aguilar, MS	Blended learning, e-learning and mobile learning in mathematics education	ZDM	2016
Spuler, M; Walter, C; Rosenstiel, W; Gerjets, P; Moeller, K; Klein, E	EEG-based prediction of cognitive workload induced by arithmetic: a step towards online adaptation in numerical learning	ZDM	2016
Gresalfi, MS	Designing to support critical engagement with statistics	ZDM	2015