A GLIMPSE INTO THE MATHEMATICS EDUCATION RESEARCH IN SINGAPORE

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Abstract

The mathematics education research projects conducted in Singapore found in the database available in the Singapore National Institute of Education are classified according to their nature. A preliminary classification of all these research projects under three types (funded research projects, doctorate thesis and master dissertation), and four categories (Theory Building, Curriculum and Policy, Classroom Teaching and Learning Practice and Teacher Professional Development) is presented. Under funded research projects, the category on classroom teaching and learning practice is the predominant category. Theory building predominates the category of research projects undertaken as part of doctorate and master studies. A glimpse into three exemplars of recent funded mathematics education research projects shows that these projects have first been identified to have addressing Classroom Teaching and Learning Practice of mathematics as their preliminary objective. Delving deeper into the description of the three studies, the three research projects are also tightly linked to Teacher Professional Development, and Curriculum and Policy components. These research projects ended with a wider dissemination to more teachers through teacher professional development and impacting the policy and the curriculum level. As demonstrated by the exemplars, categorizing the projects under the four categories is not sufficient, as these projects tackle issues of mathematics education holistically in addressing the remaining categories.

Keywords: mathematics education research projects, teaching and learning practices, professional development

INTRODUCTION

The role of education in Singapore has been evolving throughout the history of Singapore. In the early days when Singapore was under the British colonial rule, the education system served to "meet political and ethnic primordial interest" (Goh & Gopinathan, 2008). After Singapore gained independence in 1965, the politicians, led by the visionary leader Mr Lee Kwan Yew, placed much emphasis on the development of human resource, recognizing the nation's lack of natural resource after Singapore gained independence upon its separation from Malaysia. Under such a situation, the role of Singapore education system shifted to the provision of the economic development of Singapore. This prepared the newly independent nation for new economic strategies essential for survival. Today, Singapore places its emphasis on preparing the nation to strive for excellence in education, in order for the nation to be future-ready.

The nation's urge to strive for excellence in education in order to be future-ready is strongly tied to its emphasis on education research. It is a common knowledge that findings from education research is able to improve current practices in the education system. In fact, this was being affirmed by Singapore politicians. Toh, Kaur and Tay (2019) summarized part of a message by the Singapore Deputy Prime Minister (DPM) Mr Teo Chee Hean. The message was delivered during his brief closing speech in his recent visit to the National Institute of Education (NIE), the sole teacher training institute in Singapore. The DPM acknowledged that Singapore students have generally been performing well in science and mathematics according to benchmarking by international comparative studies. He specifically entrusted the education researchers of the institute with two main tasks:

(1) to understand the underlying reasons and principles of the overall good performance of Singapore students in various international comparative studies such as TIMSS and PISA; and

(2) to improve student learning, in order adapt to the future in which the conditions of the education scene can be vastly different from the current (speech by Teo, cited in Toh, Kaur and Tay, 2019).

The findings from education research could provide answer for task (1), and propose plausible solution for task (2). This conviction of the Ministry of Education is further testified by the generous funding for education research provided by the Singapore Ministry of Education (MOE) for the NIE.

Toh, Kaur and Tay (2019) provided a structure for understanding mathematics education in the recently published academic book on mathematics education in Singapore. They highlighted that any effort to understand Singapore's mathematics education should begin with the three interrelated components of mathematics education: (a) mathematics curriculum and related policy; (b) classroom enactment; and (c) teacher professional development (see Figure 1). This structure of the tripartite relationship was used to provide the structural organization of all the chapters of the academic book.

In this paper, a categorization of the mathematics education research projects that have been conducted in Singapore from 2004 onwards will first be presented. As the Singapore National Institute of Education (NIE) is the sole teacher education institute in Singapore, it is reasonable to assume that *all* mathematics education research projects conducted in Singapore have been captured in the NIE database. Transiting from broad stroke to more fine-grained details about mathematics education research projects, three large scale mathematics education research projects among the list of the official records will be used as exemplars for discussion. A trend in funded mathematics education research projects in Singapore will be presented.

THE TRIPARTITE STRUCTURAL FRAMEWORK TO UNDERSTAND MATHEMATICS EDUCATION RESEARCH IN SINGAPORE

The tripartite structural framework in Figure 1 was used to provide the overall structural organization of the handbook on mathematics education in Singapore edited by Toh, Kaur and Tay (2019). In this paper, this structure will be used for understanding mathematics education research in Singapore.

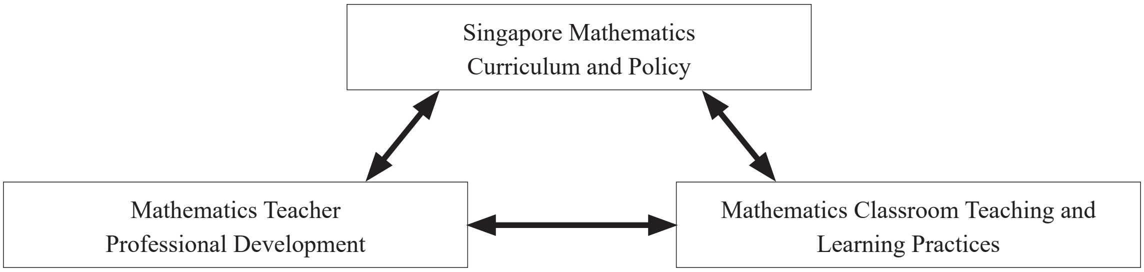

Figure 1. The three key components of mathematics education in Singapore (Toh, Kaur & Tay, 2019)

Singapore Mathematics curriculum, and policy

The Singapore mathematics curriculum consists of a general curriculum framework, which provides the general guiding principles in designing and implementing the curriculum content, and sets of mathematics syllabuses for the various mathematics subjects offered in schools for various groups of students from the primary to pre-university level. The mathematics syllabuses include specific sets of aims to guide the design and delivery of the mathematics lessons suited for the students. The guide is by no means prescriptive. It provides a useful guide to the practice and the underlying principle of the curriculum. Kaur (2019) provided a detailed discussion of the syllabus curriculum content, which will not be elaborated here.

The overall objective underpinning the Singapore school curriculum enables the nation to realize the "Desired Outcomes of Education" stipulated by the Singapore Ministry of Education (MOE). According to the Desired Outcomes, learners must be "future-ready, have a strong sense of identity and are equipped to contribute in a globalized world" (MOE, 2018).

The mathematics curriculum must also align to the various new initiatives that have been introduced into the education system from time to time. The initiative "Thinking Schools, Learning Nation (TSLN)" was introduced by the then Prime Minister Mr Goh Chok Tong (Goh, 1997). Under TSLN, Thinking Skills, Information and Communication Technologies and National Education must be incorporated as a crucial part of student learning. To align with the "Teach Less Learn More" initiative, which was first introduced by the Prime Minister in his National Day Rally speech in 2005, engaging students in the process of learning and preparing them for life skills must be realized in the national curriculum. Important ideas from the 21st century competency framework that was developed by the MOE in 2015 (Figure 2), such as to develop our students into self-directed learners, confident person, active contributor and concerned citizens, must be incorporated within the curriculum. In order for the national curriculum to align to these initiatives that have been introduced from time to time, regular revision of the national curriculum has been a regular phenomenon in the Singapore MOE. For mathematics specifically, the curriculum review is carried out once every five years.

According to the MOE (2012, 2012a), the regular mathematics curriculum review serves three key objectives to (1) ensure that the school curriculum continue to meet the needs of Singapore and the Singapore students; (2) build in students a strong foundation in the subject, and (3) make improvement in the school mathematics education (MOE, 2012, p. 9).

Figure 2. The twenty first century competency framework (MOE, 2015)

In addition to the regular curriculum review, the mathematics curriculum is also constantly benchmarked against the international trends of mathematics education to ensure that the curriculum is up-to-date and of international standard. According to the Singapore mathematics curriculum documents (MOE, 2012, 2012a), Singapore students' performance in International Association for the Evaluation of Educational Achievement's Trends in International Mathematics and Science Study (TIMSS) and OECD's Programme for International Student Assessment (PISA) have also been taken into consideration in the curriculum review process. This is not at all surprising as there is an increasing international trend that students' performance in the various international assessments (such as TIMSS and PISA) is taken as a means to measure the "excellence" (Lierse, 2018) of a country's subject-based curriculum in particular (and the country's education system in general).

Educators have pointed out that the use of the information on students' performance on these international comparative studies has also transcended beyond merely providing information about a nation's curriculum, or for educators to "peep" at the curriculum of the more successful nations. In fact, such information on students' performance has now provided a useful lens for each nation to examine how the intended mathematics curriculum is enacted in their mathematics classrooms. (Vistro-Yu & Toh, 2019). This in turn provides the Ministry of Education of each nation with useful information for important decision making in reviewing and revising the curriculum.

Although regular curriculum review has been carried out by the Singapore MOE regularly, the general curriculum framework for the Singapore mathematics curriculum (Figure 3) has been rather stable. The centrality of mathematical problem solving as the heart of the Singapore mathematics education has never been displaced or challenged since it was first introduced in the early 1990s. That problem solving is flanked by five attributes which are crucial for developing mathematical problem-solving among students: (1) Processes; (2) Skills; (3) Concepts; (4) Attitude; and (5) Metacognition, has never been denied, despite the new trends and emphasis in mathematics education around the world. Each revision of the curriculum during the review indeed strengthens the fundamental framework; it only results in minor revision to the annotation of the five attributes of the problem solving framework (Figure 3). As an illustration, aligned to the new emphasis and international trend of mathematical modelling in mathematics education, “Application and Modelling” has been incorporated as *one* of the key indicators to the “Processes” arm of problem solving.

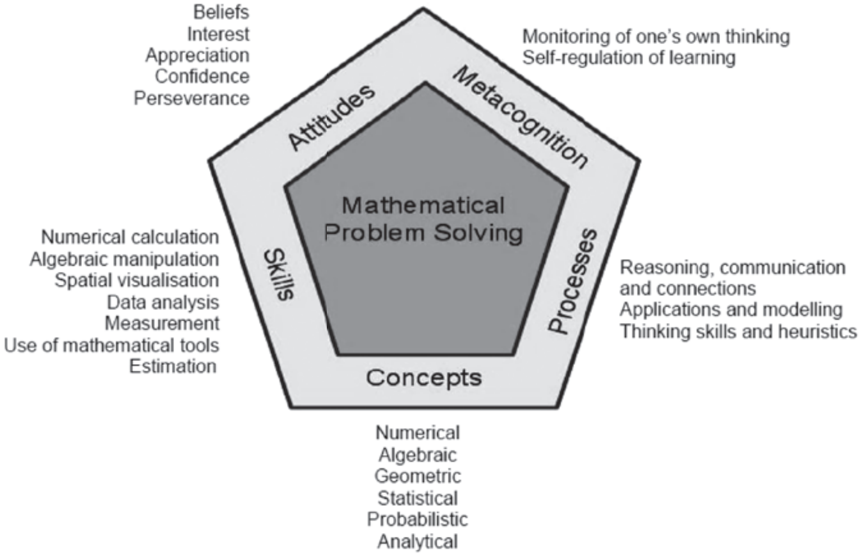

Figure 3. The Singapore Mathematics Curriculum Framework (Ministry of Education, 2012)

Mathematics classroom teaching and learning practices in mathematics classrooms

The mathematics curriculum documents reflect the designated curriculum of the Ministry of Education. The documents provide teachers with appropriate guideline for delivering the intended curriculum through the provision of the syllabuses, and teaching and learning guides. Based on these documents, classroom teachers plan their instructions, develop their own instructional material and to deliver their lessons. These practices form the *enacted curriculum* in the teachers’ classrooms. It should be noted that the curriculum documents are meant to be descriptive rather than prescriptive.

Remillard and Heck (2014) explains that, as teachers draw on the designated curriculum, which is the official curriculum document, along with other resources (instructional materials such as textbooks) to design their instruction, they create a “teacher-intended” curriculum. In creating the teacher-intended curriculum, teachers have to make substantial interpretation and decisions based on the designated curriculum. This teacher-intended curriculum, which is difficult to document, is thus translated to the enacted curriculum. The enacted curriculum can be seen as an amalgam of teachers’ teaching practices and students’ learning practices. It has been recognized as an important facet of study in mathematics education research. A study of the classroom teaching and learning practices provides useful information on the classroom teachers’ decision making processes when they deliver the teacher-intended curriculum.

Mathematics teacher professional development

Singapore emphasizes the importance of teachers’ continual professional development after they graduate from the pre-service teacher education. Educators and policy makers have regularly highlighted that graduation from pre-service teacher education is only the beginning of the path on teacher education. Continual teacher professional development is especially important in Singapore, where new national initiatives are regularly introduced into the education system. Teachers are thus expected to keep themselves updated with the latest initiatives, and build up their capacity to meet the challenges in infusing these initiatives into the curriculum.

The model of teacher professional development used by Singapore, known as the "Teacher Growth Model", was first elucidated in 2012 (MOE, 2012b). It encourages practicing teachers to look for personal professional growth through various platforms. The professional development time does not merely restrict to addressing areas that are directly related to the teachers' teaching practice. Practicing teachers are also encouraged to use part of the professional development time to learn about other areas which might not be directly related to their teaching. In this way, the teachers can be better equipped for the challenges of the unpredictable future.

Aligned to the "Teacher Growth Model", all Singapore teachers are entitled to 100 hours of professional development per year. The practicing teachers' continual professional development is fully sponsored by the MOE. Researchers such as Bautista, Wong and Gopinathan (2015) noted that the model of Singapore teacher professional development matches the criteria of "high quality" professional development that is described in the existing education literature (e.g. Avalos, 2011; Darling-Hammond & McLaughlin, 2011).

MATHEMATICS EDUCATION RESEARCH IN SINGAPORE

A glimpse into the mathematics education research projects based on the NIE database is presented in this section. The Singapore National Institute of Education (NIE) is the only institute where all education research studies (either postgraduate studies or funded education research projects) in the nation are conducted. It is thus reasonable to assume that the data reflects all significant mathematics education research conducted on mathematics education in Singapore. The education research projects that were identified as "mathematics education research" are those projects for which mathematics forms the core objective of the study, rather than being the peripheral supporting subject within the research project.

In discussing the quantity of the mathematics education research conducted in Singapore, it was decided that the number of mathematics education research projects be calculated based on the official number of research projects, rather than the number of published education research papers, for two main reasons:

(1) each research project may result in different number of publications. Hence, the number of publications may not be proportional to the number of research studies undertaken; and

(2) it is difficult to locate *all* the published papers associated with each research project, as they are published in various international or regional journals.

In this paper, all mathematics education research projects that have been captured in the database were first classified under three categories: (1) education research projects (funded by various agencies); education researches that are engaged under: (2) doctoral studies; and (3) masters studies. We used the tripartite structure (Figure 1) to classify these research projects using the three categories: (1) mathematics curriculum and policy; (2) mathematics classroom teaching and learning practice; and (3) mathematics teacher professional development.

Note that each research project may fall under more than one of the three categories (as will be illustrated below). However, the projects will be classified based on the original intention of the researchers as provided in the brief description of the research projects and the abstract of each of the postgraduate doctorate or master thesis.

Funded research projects and postgraduate research

Among the many funded education research projects in the NIE database, there were 45 that could be classified as mathematics education research projects. In classifying these mathematics research projects using the tripartite structure, the need to create an additional category to exhaust the classification of all the projects was decided. In other words, the tripartite structure (Figure 1) was not sufficient for the classification purpose. The additional category that was created was termed as "Theory Building". We used an exemplar of a research project to illustrate each of the four categories.

Policy and curriculum: This category of research projects is exemplified by a research project entitled "Secondary Analyses of Teacher Education and Development Study in Mathematics" which was conducted between 2014 and 2017. According to the description of this research project,

> The aim of this new project is to gain further knowledge and insights into mathematics teacher education in Singapore and overseas institutions through secondary analyses of the data collected in the international TEDS-M project (Teacher Education and Development Study in Mathematics).

This project aimed to gather information about mathematics teacher education in Singapore. This project as a part of a larger international comparative study allows each participating country (in particular, Singapore) to learn the best practices of all participating countries. In particular, the findings of this project offered insight into the local curriculum and policy matters.

Professional development: This category is exemplified by a research project entitled "Enhancing the Pedagogy of Mathematics Teachers to Facilitate the Development of 21st Century Competencies in their Classrooms" that was conducted in the period 2014 to 2017. According to the description of this project,

> The project primarily aims to engage a group of in-service secondary school Math teachers and develop a teaching strategy to facilitate the development of 21st century competencies in Math classrooms.

This project's primary goal was to build practicing teachers' capacity in effective teaching in school classrooms. It proposed long-term collaboration between the researchers and the practicing teachers. This mode of teacher professional development deviates from the usual workshop mode of professional development that assumes the deficit model of teacher capacity.

Teaching and learning practices: This category of research projects primarily begins with studying the classroom enactment as the primary aim. This category of projects is exemplified by a mathematics education research project entitled "MAthematics is Great: I Can And Like (MAGICAL)". According to the project description,

> This project aims to develop a package of alternative approach to teach Lower Secondary Normal (Technical) mathematics using storytelling, comics, and other graphic stimulus in context. It will study the effect of this alternative approach on students' mathematical self-concept, motivation to learn mathematics and achievement in mathematics.

Judging from the description, the primary objective of the project was to study the enactment in the mathematics classroom using the researchers' new approach of using comics and storytelling for low ability students. The projects which were classified under the category "Teaching and Learning Practices" included those that introduced novel intervention strategies in classroom practice, and a study of the impact of these interventions on student learning.

Theory building: Theory building can be explained as "the purposeful process or recurring cycle by which coherent descriptions, explanations, and representations of observed or experienced phenomena are generated, verified and refined" (Lynham, 2000, p. 161). This category of research projects covers those in which the researchers attempted to develop the "theory" underlying a set of phenomena in education. An exemplar of this category of project was one which was entitled "Portraits of Teacher Noticing during Orchestration of Learning Experiences in the Mathematics Classrooms". According to the project description,

> This project has two main goals. First, it involves developing a local theory to describe, and prescribe, what and how exemplary teachers notice when they orchestrate Learning Experiences in their classrooms. Second, it is aimed at designing a toolkit that can be used by teachers to promote students' thinking through high-quality Learning Experiences.

Table 1. Classification of mathematics education research conducted in Singapore since 2000

Category \ Types	Funded Math Education Research projects	Doctorate students' research	Master students' research
Policy & curriculum	12 (26.7%)	2 (8.8%)	13 (12.5%)
Professional development	15 (33.3%)	1 (4.4%)	1 (1.0%)
Teaching & Learning Practices	17 (37.7%)	5 (21.6%)	38 (36.5%)
Theory building	1 (2.3%)	15 (65.2%)	52 (50.0%)
Total	45	23	104

We divided all the research projects under three *types* of mathematics education research (funded research projects, doctorate studies and masters studies). Under each type of research projects, we classified them using the four categories: Policy and curriculum; Professional development; Teaching and learning practices; and Theory building. A breakdown of all the research projects under the types and categories is shown in Table 1.

Under the category "funded mathematics education research projects", the most frequent category of research is "Classroom Teaching and Learning Practices" (37.7%), while the least frequent is "Theory Building" (2.3%). On the other hand, the most frequent category of research for postgraduate research (including both master and doctorate level studies) level is on “Theory Building” (65.2% for doctorate and 50% for master level studies), while the least common category of education research is “Professional Development” (4.4% for doctorate and 1.0% for postgraduate research).

The above data suggests that different types of mathematics education research serve different functions. Undeniably, the priority of postgraduate studies is to engage candidates to experience the process of research, under which theory building is an essential component. Thus, relatively many research projects from postgraduate studies have their primary goal as contributing towards the building of theory about specific aspects of mathematics education. On the other hand, the funded education research projects conducted in NIE is geared towards improving teachers’ effectiveness (Teacher Professional Development, 33.3%) for a more efficient enactment of the curriculum in the authentic classroom (Teaching and Learning Practice, 37.7%).

THREE EXEMPLARS OF MATHEMATICS EDUCATION RESEARCH PROJECTS

In this section, three exemplars of the funded research projects from the NIE database are discussed with greater detail. The discussion includes the implication of these mathematics education research on the schools, curriculum and teacher professional development.

EXEMPLAR ONE (TWO PARTS): MATHEMATICAL PROBLEM SOLVING IN SECONDARY SCHOOL CLASSROOMS

The first exemplar (which consists of two research projects in the database, or two *parts* to the project) reported here is a study on an enactment of mathematical problem solving in the Singapore mathematics classroom. The first part, which was entitled MProSE (Mathematical Problem Solving for Everyone), was first conducted in one Singapore school beginning in 2008. The second part, entitled MInD (MProSE: Infusion and Diffusion), was a scaling up of the first part which commenced in 2012 after the completion of MProSE. The objective of the two parts of the project was to enact mathematical problem solving in the truest spirit of problem solving in the Singapore mathematics classrooms.

The first part of the project on enacting mathematical problem solving was initiated to address the observation among the local mathematics educators that mathematical problem solving had not been enacted according to the underlying intent of problem solving in many Singapore mathematics classrooms. The researchers proposed this project based on a combination of their classroom experience, anecdotal evidence in Singapore classrooms and findings from education literature on the feasibility of enacting authentic problem solving in the mathematics classrooms. Local studies on mathematical problem solving in Singapore up to 2009 mainly focused on theoretical aspects of mathematical problem solving. It was also noted that mathematical problem solving was not a common practice in the Singapore mathematics classrooms (Foong, 2009, cited in Toh et. al., 2019). Some studies have also shown that Singapore students generally were not well prepared in handling unseen mathematics problems (Dong et al., 2002, Kaur, 2009), although overall

Singapore students' performance in TIMSS and PISA has been very good comparatively to many other participating countries.

The main objective of the two parts of the research project MProSE and MInD was not building theory about problem solving. Its main goal was to translate the already existent problem solving theories into classroom practice (i.e. enacting authentic problem solving in the mathematics classrooms) and to study the impact of authentic problem solving on students' learning of mathematics. This objective is aligned to Alan Schoenfeld's (2007) advocate in the 2007 special issue on problem solving in the ZDM journal that the current focus of problem solving should not lie in theory building, as "[t]he theory has been worked out" (Schoenfeld, 2007). What was more pressing for the mathematics education community for work to be done should be to "translate decades of theory building about problem solving into workable practices in the classrooms" (Schoenfeld, 2007, cited in Toh et al., 2019)

Phase one: Enactment [and conceptualization]

Each past consisted of phases. The first phase of MProSE was initiated for implementation in one Singapore secondary school. The first phase of MInD started in five Singapore mainstream schools. The first phase involved the conceptualization of how mathematical problem solving should be enacted in the Singapore mathematics classroom according to the true spirit of problem solving. The initial conceptualization of the researchers' approach to teach problem solving was recorded in Toh, Quek and Tay (2008). The researchers believed that generally students were resistant to following Polya's problem solving model (the model which has been emphasized in the Singapore mathematics curriculum). Even the higher ability students, who could solve challenging mathematics problems, refused to expend more effort to check their solutions or to extend or generalize the problems that they had managed to solve successfully. In other words, most students were resistant to be engaged in the metacognitive part of the problem solving process. The MProSE researchers modelled their teaching approach after the suggestions of Schoenfeld (1985), who suggested to explicitly voice out specific control questions at regular intervals so as to make students to pause at appropriate juncture and be more mindful of their own thinking in their solution of the problems.

The researchers translated the problem solving approach that they advocated into executable classroom episodes to engage students to go through the complete problem solving process (this is especially critical when the students encountered blockage and were struggling in solving the problem). To achieve this, the researchers constructed a scaffolding worksheet resembling a science practical worksheet that is used in science practical lessons. According to the researchers' proposal, such specialized problem solving lessons should be treated as a mathematics 'practical' lessons, in the same way that science practical lessons correspond to the science subjects. The researchers' goal was to achieve a paradigm shift in the way students looked at these 'difficult, unrelated' problems which had to be done in this 'special' problem solving class. The details of the research rationale and design, as well as the enactment of the problem solving classes are described in Toh, Quek, Leong, Dindyal and Tay (2011).

Schroeder and Lester (1989) distinguished three forms of problem solving in relation to the school curriculum, and this classification is still widely used in the mathematics education literature (e.g. Ho & Hedberg, 2005; Stacey, 2005)

- Teaching *for* problem solving

- Teaching *about* problem solving
- Teaching *through* problem solving

The research team felt that the missing link in the mainstream school curriculum was that of Teaching *about* problem solving, which could be filled by their suggestion of the 'special' problem solving lessons developed in MProSE and MInD. In the proposed problem solving lessons, the students must first be acquainted with the mathematical vocabulary and processes of problem solving, and given the opportunity to apply these in solving mathematics problems. In other words, problem solving is foregrounded, while the mathematics content forms the background context for students to learn and experience problem solving. In the proposed problem solving lessons, the teachers were required to model and teach explicitly to students the language and strategies used in problem solving, and to provide appropriate scaffolding when students were 'stucked' at the problems. Thus, the role of the teacher in the specialized problem solving lessons is more to facilitate developing students' problem solving skills, rather than explaining mathematical concepts as a classroom teacher. The mathematics practical worksheet was used in the lessons to facilitate student engagement in problem solving. Full detail of the lesson enactment can be found in Toh, Quek, Leong, Dindyal and Tay (2011), and an online version of the proposed MProSE problem solving lessons can be found in http://math.nie.edu.sg/mprose.

MProSE and MInD generally show that students (inclusive of high ability, middle ability and low ability) responded positively to the MProSE proposed approach to learning mathematical problem solving. In addition, Leong et al (2011) reported that the teachers benefitted from the new problem solving approach. In particular, the teachers benefitted from the close partnership between the researchers and the school teachers in enacting such a problem solving lessons and their use of a modified version of lesson study to build up teachers' capacity.

Phase two: Teacher professional development

The empirical data of MProSE and MInD suggested the feasibility to engage students in the authentic mathematical problem solving in Singapore mainstream schools. The success of enacting problem solving greatly hinged on the capacity of the mathematics teachers. This led the researchers to turn to the mission of building teacher capacity for the sustainability of problem solving.

The researchers infused the MProSE problem solving approach (inclusive of the scaffolding practical worksheet, use of special problem solving lessons, Polya's model as the underpinning theoretical framework) into the teaching of undergraduate mathematics content courses in Introductory Number Theory and Differential Equations (DE) courses (Toh et al., 2013, Toh et al., 2014) in NIE. This not only enabled more student teachers to be exposed to this innovative method prior to graduation from NIE; it also allowed the pre-service teachers to apply the problem solving approach for their own learning of (undergraduate) mathematics. Toh et al. (2013) and Toh et al. (2014) reported a gain in student learning in both paper.

A curriculum review after this pilot study in the mathematics teacher education programme in NIE eventually led to the development of a new course on Mathematical Problem Solving (MPS) module, which was made mandatory for all pre-service mathematics teachers at Year One undergraduate level (Toh et al, 2019, pp. 156-158). The MPS module introduced the student teachers the Polya's problem solving model, and how the MProSE/MInD problem solving approach could be applied to solve challenging undergraduate mathematics problems (from various branches of undergraduate mathematics).

The student teachers had already been exposed to problem solving prior to the introduction of the MPS module in the teacher education programme. Within the mathematics pedagogy courses, which are mandatory for all mathematics student teachers, all student teachers would have already been introduced to Polya's problem solving model and had been engaged in solving "challenging" mathematics problems using the model (see for example, Kaur and Toh, 2011). In the mathematics pedagogy courses, however, the students approached problem solving from the perspective of teachers and the "challenging" problems that they were engaged were secondary school mathematics problems. Those "challenging" problems were not authentic problems for adults. Thus, the MPS course complemented the pedagogy course by engaging the student teachers with hands-on engagement on *authentic* problem solving on the problems which were challenging to the student teachers. To date, anecdotal evidence suggests that student teachers who have been exposed to MPS module are able to exercise their "control" (Schoenfeld, 1985) better in problem solving, and tend to be able to extend and generalize a mathematics problem that they have solved. This is in agreement to our earlier findings in Toh et al. (2013).

Further development of the research

The objective of MProSE and MInD was to enact mathematical problem solving in the secondary school mathematics classrooms. MProSE and MInD have resulted in two undergraduate students in attempting to conceptualize the MProSE approach of problem solving at the upper primary level (Liang & Toh, 2018; Yong & Toh, 2019). Liang and Toh (2018), and Yong and Toh (2019) tweaked the MProSE/MInD design by selecting appropriate mathematical tasks for upper primary students. They further modified the scaffolding "practical" worksheet to meet the needs of the upper primary school students, as the worksheet was too wordy and unappealing, hence cognitively heavy for young children at the primary level.

MProSE and MInD have also resulted in the conceptualization of the *processes* and *product* of assessing mathematical problem solving (Toh et al., 2011a). The entire process of teaching and assessing problem solving proposed by the research has captured the attention of the Singapore MOE. In the most recent curriculum review for the new Pre-University mathematics (the H3 mathematics), elements of MProSE problem solving was incorporated into the syllabus document. MProSE and MInD have also resulted in the development of several in-service teacher professional development courses and, in fact, have influenced the development of several other approaches of problem solving in the mathematics classrooms.

Problem solving and the tripartite structure

MProSE began and MInD continued with the goal of enacting problem solving in the mathematics classroom according to the underlying intent of problem solving. The researchers were mindful of building teacher capacity through professional development in order to ensure the sustainability of the MProSE/MInD approach. With the researchers' conviction of the efficacy of the researched approach of teaching problem solving, this approach was introduced to pre-service teacher education programme in both the pedagogy course and the undergraduate content course. Several in-service teacher professional courses were also developed based on the MProSE/MInD approach. In this way, the researchers targeted to build a larger pool of teachers who would be able to use this approach to teach problem solving.

Almost simultaneously, the MProSE/MInD problem solving approach inspired the curriculum review of one mathematics subject at the pre-university level (the H3 Mathematics). The revised curriculum of H3 Mathematics has a significant portion on problem solving. This is the first substantial effort to build up a larger pool of students who are able to do authentic problem solving. The tripartite framework of Figure 1 can be interpreted as shown in Figure 4 for this problem solving research project.

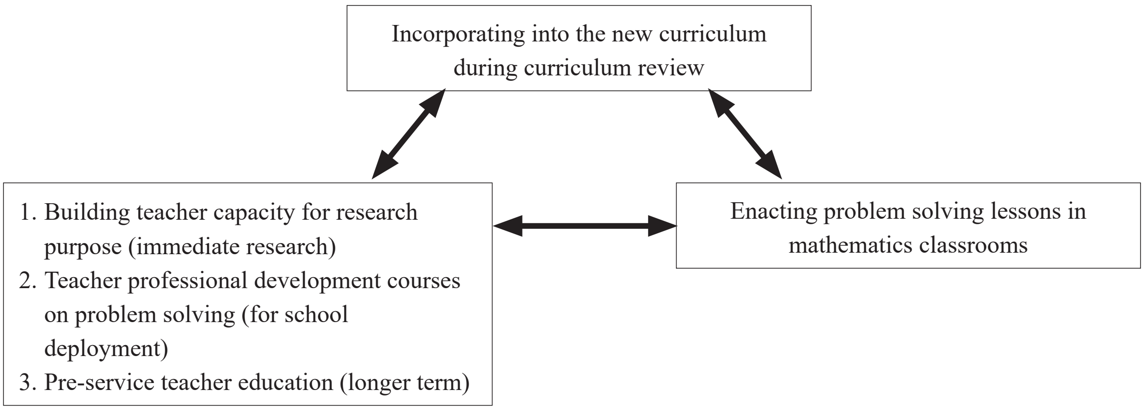

Figure 4. The tripartite relationship realized in the problem solving project

EXEMPLAR TWO: COMICS FOR MATHEMATICS INSTRUCTION

The research project on using comics for mathematics instruction (in the secondary classrooms) arose out of a concern among the educators and policy makers in Singapore on the existence of a large group of low attaining mathematics students. The group of low attaining students in Singapore who did not perform well in TIMSS was significantly larger in comparison with the low attaining students in the East Asian counterparts (Hong Kong, South Korea, Taiwan and Shanghai), despite the fact that the overall performance of Singapore in TIMSS was comparable to the East Asian counterparts (Toh & Kaur, 2019).

Based on a survey conducted by Toh and Lui (2014), there was concern among local mathematics teachers about mathematics learning by the low attaining students. The teachers participating in the survey concurred that the existing official instructional material was inadequate for this group of students who needed more help. In response to this, many teachers had already begun developing their own creative resource to teach the low attaining students. One of the approach that was mentioned by the teachers was the use of cartoons and comics, combined with humor, in teaching mathematics. This finding led to the conceptualization of the research project on using comics to teaching mathematics to low attaining students. The first part of this project was entitled MAGICAL (MAthematics is Great: I Can And Like), and its subsequent scaling up project was called SUPERMAGICAL (Scaling UP of Education Research MAGICAL). For brevity, we shall term both parts of the project MAGICAL in the subsequent discussion.

As reflected in the title of the project, the research aims to address both the cognitive ("can") and the affective ("like") aspects of student learning. The researchers conducted extensive literature review and found that contextualizing mathematics has the potential of increasing students' motivation for learning and their engagement with the entire process of learning (e.g. Cordova & Lepper, 1996). There have already been papers published on research carried out in the other parts of the world on using comics for classroom instruction. However, little is done on how a comics mathematics lesson could be enacted, or the impact of comics on student learning.

In the above studies, the term "*comics*" was used to refer to an approach of conveying messages in the form of stories and which make use of imaginaries, usually in the form of cartoons. As an illustration, a portion of a comic strip that was developed in MAGICAL for the teaching of Percentage to lower secondary (Grades 7 and 8) students is shown in Figure 5. Interested readers may refer to the website http://math.nie.edu.sg/magical for more comic strips that were used in the teaching of Percentage.

Figure 5. A portion of a comic strip on Percentage (http://math.nie.edu.sg/magical)

Design process of the comics instructional package

The goal of MAGICAL was to study the feasibility of using comics for mathematics instruction, and its impact of comics on students' learning of mathematics. The researchers developed complete comics instructional package based on the curriculum requirement for several chapters selected from the lower secondary mathematics curriculum. The researchers proposed the package to be used as "replacement unit" (Leong, et al., 2016). As a "replacement unit", the teachers participating in the research used these packages to *replace* the usual resources comprising of textbooks and other instructional material when they taught these selected chapters. The researchers also provided the teachers with suggested lesson proposals in enacting the mathematics lessons using comics. In the lesson proposals, the researchers adopted storytelling pedagogy for conducting the classroom lessons. Toh, Cheng, Jiang and Lim (2016) provided a detailed description of the approach. The participating teachers were urged to adhere to the lesson proposals as closely as possible. However, they were given the liberty to tweak the lessons to meet the learning needs of their students based on their professional judgement. The lessons were video-recorded and analysed by the researchers.

Teacher professional development while engaging in the research project

As the above approach was unprecedented, professional development became an indispensable part of the research project for the participating teachers who were required to deliver the comics instructional package. The professional development consisted of detailed discussion between the researchers and the participating teachers on the underlying intent and principles of comics for mathematics instruction. The researchers further demonstrated how storytelling could be used in a mathematics lessons, how visual cues could be used to develop mathematical literacy and civic mindedness, and also how the underlying mathematics concepts could be elicited through storytelling. The researchers further worked with the participating teachers on refining the package to better meet the needs of the students after the lesson delivery. Special note on how teachers made tweaks to the package was taken into consideration in refining the package.

MAGICAL was carried out in three Singapore mainstream secondary schools. The project was subsequently scaled up under SUPERMAGICAL to seven mainstream secondary schools and one primary school. The process of adapting the comics instructional package was a challenge to the researchers as the students' capacity varied greatly across different school. The second challenge encountered by the researchers was the alignment of the instructional package, which had been originally designed for the lower secondary mathematics curriculum, to the upper primary mathematics curriculum. The process of refinement of the comics instructional package in the individual school continued for several iterations, according to the original spirit of design experiment. The refinement process is summarized in Figure 6.

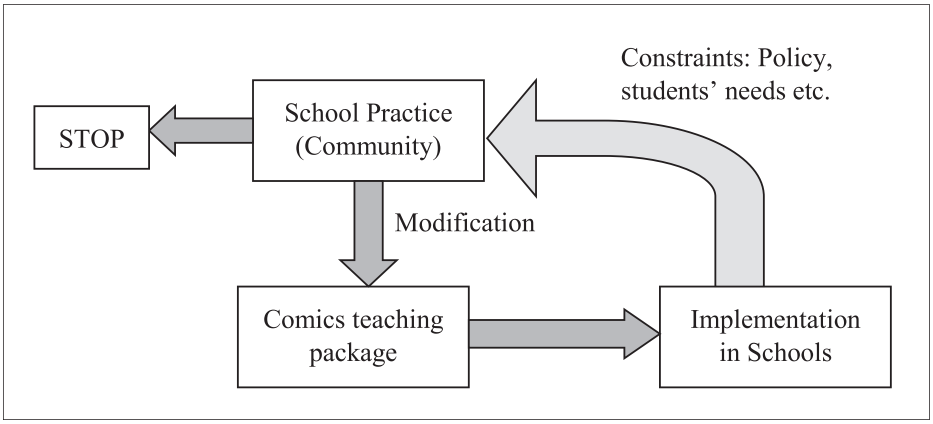

Figure 6. The process of implementation of the comics teaching package

Instructional material and the curriculum

MAGICAL, at its early stage with some preliminary findings, had resulted in some ideas of comics for mathematics instruction being incorporated into one series of mathematics textbooks for the low attaining students in Singapore. In this series of textbooks, each chapter contained at least one comic strip within the chapter content and one comic strip as the chapter opener. This was an attempt to convey one overarching mathematical idea and one main mathematics concept in a less traditional and humorous way. The concept of using comics instructional package as a replacement unit had not been developed at that time when the textbook was developed. In this sense, the comics research projects had impacted the process of textbook development at its infant stage.

For wider dissemination to more Singapore teachers, the process of using comics for classroom instruction, its rationale and the approach have been disseminated to Singapore teachers through various platforms, such as teachers' workshop and sharing sessions (e.g. Toh & Kaur, 2019). More recently, a symposium sharing the challenges and success stories of using comics for mathematics instruction by both researchers and the participating teachers was conducted (the detail of the symposium can be found in https://www.nie.edu.sg/niews/september2019/magical-bringing-mathematics-to-life-through-comics.html). More articles in both academic and professional journals will be published for wider dissemination of the findings of MAGICAL in the future.

The tripartite relationship of Figure 1 as interpreted for MAGICAL as in Figure 7 below.

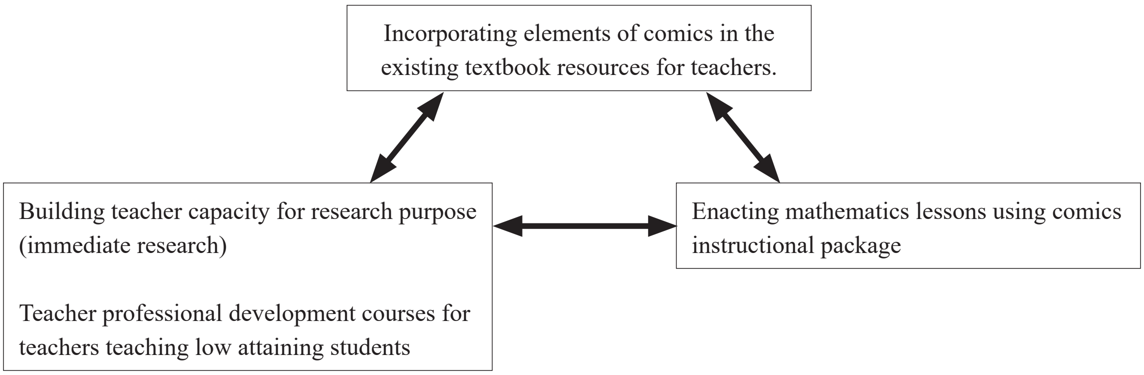

Figure 7. The tripartite relationship as realized in the comics research projects

EXEMPLAR THREE: ENACTMENT OF SCHOOL CURRICULUM IN THE SINGAPORE MATHEMATICS CLASSROOM

This research project was initiated by a group of mathematics education researchers from NIE to have direct knowledge about what happens in an authentic Singapore mathematics classroom. The project began with obtaining snapshots of classroom practices of 30 experienced and competent mathematics teachers, followed by a survey on the prevalence of the exemplary practices among *all* Singapore mathematics teachers. This project can be seen as a response to some early research findings about the teaching practices in Singapore mathematics classrooms by other researchers.

In an earlier research led by Professor Hogan on understanding the mathematics instruction in the Singapore mathematics classrooms (Hogan, Towndrow, Chan, Kwek, & Rahim, 2013), it was found that many Singapore teachers placed much emphasis on procedural knowledge at the expense of conceptual development of their students. The study also found that teachers spent much curriculum time on "performative tasks" relative to "knowledge building tasks" (Hogan et al., 2013). In other words, Singapore teachers tended to focus more on skill development during lessons over relational understanding of the related concepts (Hogan et al., 2013; Kaur, 2009).

However, such observations were unable to explain Singapore students' stellar performance in PISA 2012 and PISA 2015. As it is well known among the mathematics education community, PISA questions require students to be proficient in mathematical reasoning and knowledge building rather than mere procedural fluency. This apparent discrepancy led a group of mathematics education researchers in NIE to conceptualize the this project to study what exactly happens in the typical Singapore mathematics classrooms (Kaur, Wong & Toh, 2017).

Two phases of the study

The study consisted of two phases. The first phase began with a study of how 30 experienced and competent mathematics teachers enacted the curriculum in their mathematics classroom. The researchers reached a consensus on an operational definition of an "experienced and competent mathematics teacher" as one who has more than five years of uninterrupted teaching of mathematics up to the time the research was conducted, and that the education community recognizes him or her as a 'good' mathematics teacher. With the consent of the identified experienced and competent teachers who had agreed to participate in the study, the researchers attended the lessons of the participating teachers on one particular mathematics subtopic, video-recorded the complete set of lessons. The Complementary Accounts Methodology, which was first proposed by Clarke (1998, 2001) and widely used in the study of classrooms internationally, was used in the data collection. The participating teachers and selected students from the video-recorded classes were interviewed. The instructional material developed and used by the teachers were collected and analysed by the researchers.

The researchers identified exemplary pedagogical practices of the 30 experienced and competent teachers and the strategies that they had used to design their instructional materials. The Singapore mathematics curriculum framework (Figure 3) was used as the theoretical framework for this study. In the second phase of the research, a survey was crafted based on the exemplary pedagogical practices and the design of instructional materials. The survey was administered to more than 600 Singapore mathematics teachers on how prevalent the practices of the experienced mathematics teachers were found in the Singapore mathematics classrooms in general.

Some findings from the study

Kaur, Wong and Toh (2017) reported some preliminary finding from an analysis of two lessons of a participating competent teacher using the Schoenfeld's Teaching for Robust Understanding (TRU) framework (Schoenfeld, Floden, and the Algebra Teaching Study and Mathematics Assessment Project, 2014). Their report only utilized one of the five dimensions (that of mathematics content) of a Mathematically Powerful Classroom using the TRU framework. It was found that all the indicators of the Mathematics Content dimension were present in the lessons of the participating teachers. In a case study of one of the experienced and competent teachers participating in this study, the research team (Leong, Cheng, Toh, Kaur & Toh, 2019) found through the interview with one of the experienced teachers that in designing his own teaching material, the teacher consciously attempted to make all teaching points explicit for his students.

An important finding in this research that was reported in Kaur et al. (2019a) is the commonality among all the lessons of the experienced teachers reflect the presence of an instructional core, which was termed by Kaur et al. (2019a) as the "DNA" of the Singapore mathematics lessons. Based on the study, the lessons of the experienced and competent teachers were found to consist of several cycles, the DSR (Development – Student Work – Review of student work) cycle. Each classroom lesson consisted of several instructional objectives that the teachers attempted to achieve. Each DSR cycle was developed by the teacher to achieve an instructional objective. Each lesson typically consisted of several DSR cycles. In development the teacher shows, tells, explains, guides or demonstrates new skills. This is followed by Student work consisting of the classwork or homework assigned by the teachers, which is then followed by Review of students' work. It was also seen that there could be loop in the S-R link for the teachers to ensure that their students have fully met the instructional objective of this particular DSR cycle before the teachers moving on to the next instructional objective.

Hence, what was typically dismissed as traditional drill-and-practice in the Singapore classroom in fact consisted of intricate DSR cycles that ensure that all the students had achieved deep learning of the related mathematics knowledge. The next phase of the project in analyzing the prevalence of the practices of the pedagogical practices of the experienced mathematics teachers among the wider education community is still underway. The results will be reported sometime in the future.

Teacher professional development and impact on the curriculum

The preliminary findings of the enactment project have been compiled into a teaching package consisting of 12 questions on mathematics teaching to be disseminated to all Singapore teachers through a symposium (Kaur, et al, 2019a). The findings will also result in new teacher professional development courses for Singapore mathematics teachers, and new standards of mathematics education research in the Singapore schools. The findings of this study will further provide data for the Singapore MOE in their regular mathematics curriculum review to make informed decisions based on research on Singapore schools. This project is not an intervention project. It began with the researchers' interest in understanding Singapore mathematics classroom teaching and learning practices. The tripartite relationship of this project can be summarized in Figure 8 below.

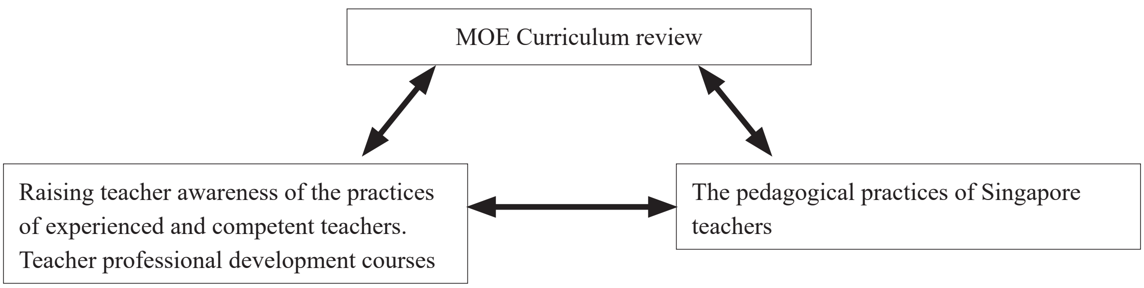

Figure 8. The tripartite relationship as realized in the enactment project

DISCUSSION AND CONCLUSION

All the mathematics education research projects conducted in Singapore can be broadly classified under three types: (1) funded research projects and projects that arose out of postgraduate studies, either (2) at the Masters level or (3) the doctorate level. The research projects can also be subdivided into four main categories: (a) Policy and curriculum; (b) Professional development; (c) Teaching and learning practice; and (d) Theory building. A matrix of the three types versus the four categories is tabulated in Table 1.

Table 1 shows that under each of the postgraduate studies, the category of theory building predominates all the research projects. This aligns to the key objectives of postgraduate studies to train the candidates to build and test their own theory about education. Under funded mathematics education research, teaching and learning practice is dominant. This is attributable to the pragmatic approach to research by the funding agency (mainly MOE). The MOE is primarily concerned with the positive impact that research projects can bring in the mathematics classrooms.

By using the above three exemplars of funded research projects for discussion, we observe that such funded projects tend to take classroom teaching and learning practice as the concern, in either introducing an intervention of a new approach of teaching (the first two exemplars); or to address a biased view about the Singapore mathematics classroom (exemplar three). However, it turns out that, to ensure sustainability of intervention or wider dissemination of information and good practices, these projects address the professional development of teachers beyond the teachers participating in the research projects. The findings of the study tend to have impact on the curriculum and policy level (the first and third exemplars impacted policies and curriculum; the second exemplar impacted the textbooks). This illustrates the close connection across the three components (Curriculum and Policy; Teacher Professional Development and Classroom Teaching and Learning Practices) in the tripartite structure of Figure 1, and is the characteristic feature of funded mathematics education research in Singapore.

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